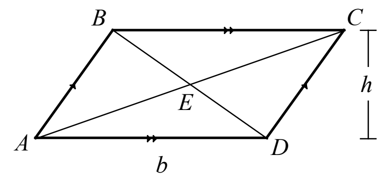
CHAPTER 18
Polygons and Quadrilaterals

18-1. Parallelograms

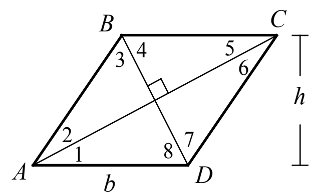


A **parallelogram** (▱) is a quadrilateral with two pairs of parallel opposite sides.

In ▱$ABCD$, $\overline{AB} \parallel \overline{CD}$ and $\overline{BC} \parallel \overline{AD}$.

Properties of Parallelogram

Opposite sides are congruent.	$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{AD}$
Opposite angles are congruent.	$\angle BAD \cong \angle BCD$ and $\angle ABC \cong \angle ADC$
Consecutive angles are supplementary.	$m\angle ABC + m\angle BAD = 180$ and $m\angle ADC + m\angle BCD = 180$
The diagonals bisect each other.	$AE = CE$ and $BE = DE$



A **rhombus** is a parallelogram with four sides of equal measure. The diagonals of a rhombus are perpendicular to each other, and each diagonal of a rhombus bisects a pair of opposite angles.

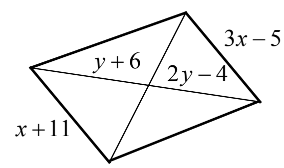
In rhombus $ABCD$, $AB = BC = CD = DA$, $AC \perp BD$, $m\angle 1 = m\angle 2 = m\angle 5 = m\angle 6$, and $m\angle 3 = m\angle 4 = m\angle 7 = m\angle 8$.

Theorem

The area of a parallelogram equals the product of a base and the height to that base. $\boldsymbol{A = b \cdot h}$

The area of a rhombus is half the product of the lengths of its diagonals (d_1 and d_2). $\boldsymbol{A = \frac{1}{2} d_1 \cdot d_2}$

Example 1 □ Find the values of the variables in the parallelogram shown at the right.



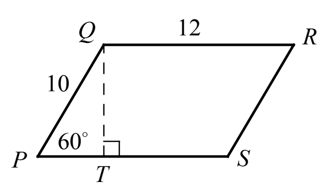
Solution □ $x + 11 = 3x - 5$ — Opposite sides of ▱ are ≅.

$16 = 2x \Rightarrow x = 8$

$y + 6 = 2y - 4$ — The diagonals of ▱ bisect each other.

$y = 10$

Example 2 □ Find the area of parallelogram $PQRS$ shown at the right.



Solution □ Notice that ΔPQT is a 30°-60°-90° triangle.

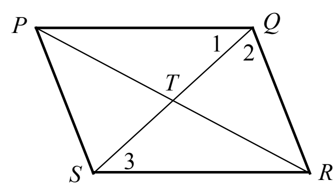
$PT = \frac{1}{2} PQ = \frac{1}{2}(10) = 5$

$QT = \sqrt{3} PT = \sqrt{3}(5) = 5\sqrt{3}$

Area of $PQRS = b \cdot h = 12 \cdot 5\sqrt{3} = 60\sqrt{3}$

Exercise - Parallelograms

Questions 1-5 refer to the following information.



In $\square PQRS$ above, $PT = x + 2y$, $ST = 8x - y$, $PR = 32$, $TQ = 26$, $m\angle 1 = 6a$, $m\angle 2 = 10a$, $m\angle 3 = a^2 - 7$ and $m\angle PRS = 4a$.

1

What is the value of x?

2

What is the value of y?

3

What is the measure of $\angle PQR$?

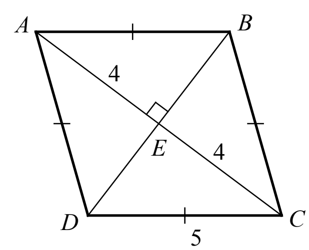
4

What is the measure of $\angle QRS$?

5

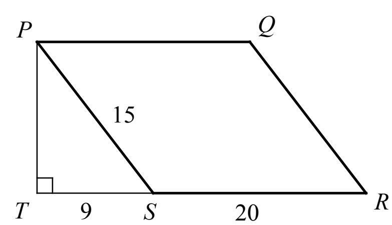
What is the measure of $\angle QTR$?

6



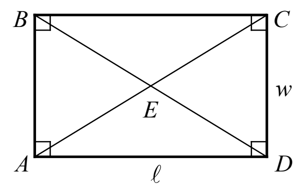
What is the area of rhombus $ABCD$ above?

7



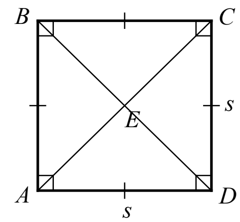
In the figure above, $PQRS$ is a parallelogram and PTS is a right triangle. What is the area of the parallelogram $PQRS$?

18-2. Rectangles, Squares, and Trapezoids



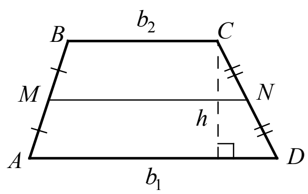
A **rectangle** is a quadrilateral with four right angles. The diagonals of a rectangle are congruent and bisect each other. The diagonals divide the rectangle into four triangles of equal area.

In rectangle $ABCD$, $AE = BE = CE = DE$.
Area of $\Delta ABE =$ Area of $\Delta BCE =$ Area of $\Delta CDE =$ Area of ΔDAE



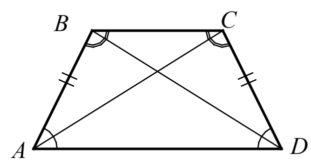
If a quadrilateral is both a rhombus and a rectangle, it is a **square**. A square has four right angles and four congruent sides. The diagonals of a square are congruent and bisect each other.

In square $ABCD$, $AB = BC = CD = DA$, $\overline{AB} \perp \overline{BC} \perp \overline{CD} \perp \overline{DA}$, and $AE = CE = BE = DE$.



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

The **midsegment** of a trapezoid is parallel to the bases and the length of the midsegment is the average of the lengths of the bases. Trapezoid $ABCD$ with median $\overline{MN}$, $\overline{AD} \parallel \overline{MN} \parallel \overline{BC}$ and $MN = \frac{1}{2}(b_1 + b_2)$.



If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**. The diagonals of an isosceles trapezoid are congruent. Each pair of base angles of an isosceles trapezoid is congruent. For isosceles trapezoid $ABCD$ at the right,

$AC = BD$, $m\angle BAD = m\angle CDA$, and $m\angle ABC = m\angle BCD$.

Theorems - Areas of Rectangle, Square, and Trapezoid

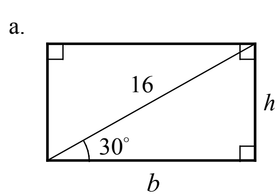
The area of a rectangle is the product of its base and height. $A = b \cdot h$

The area of a square is the square of the length of a side. $A = s^2$

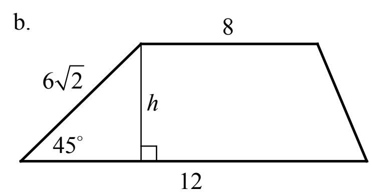
The area of a trapezoid is half the product of its height and sum of the bases. $A = \frac{1}{2}h(b_1 + b_2)$

Example 1 □ Find the areas of the quadrilaterals shown below.

a.



b.



Solution □ a. The quadrilateral is a rectangle.

$h = \frac{1}{2}(16) = 8$, $b = h \cdot \sqrt{3} = 8\sqrt{3}$ — Use the $30°$-$60°$-$90°$ Δ ratio.

$A = b \cdot h = 8\sqrt{3} \cdot 8 = 64\sqrt{3}$ — Area formula for rectangle

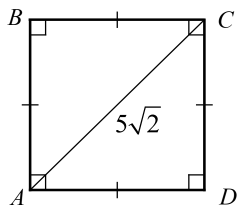
b. The quadrilateral is a trapezoid.

$h \cdot \sqrt{2} = 6\sqrt{2} \Rightarrow h = 6$ — Use the $45°$-$45°$-$90°$ Δ ratio.

$A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2}(6)(8 + 12) = 60$ — Area formula for trapezoid

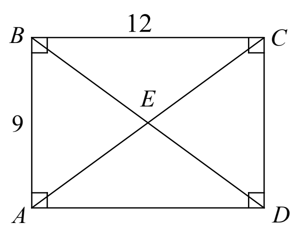
Exercise - Rectangles, Squares, and Trapezoids

1



In square $ABCD$ above, the length of diagonal AC is $5\sqrt{2}$. What is the area of the square?

Questions 2 and 3 refer to the following information.



In the figure above, $ABCD$ is a rectangle.

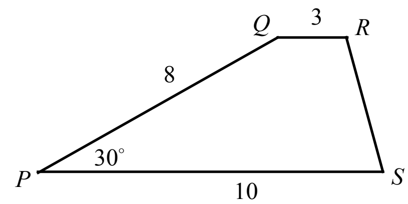
2

What is the length of AE?

3

What is the area of ΔCED?

4



What is the area of trapezoid $PQRS$ above?

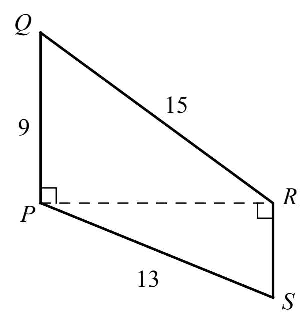
A) 20

B) 24

C) 26

D) 32

5



What is the area of trapezoid $PQRS$ above?

A) 64

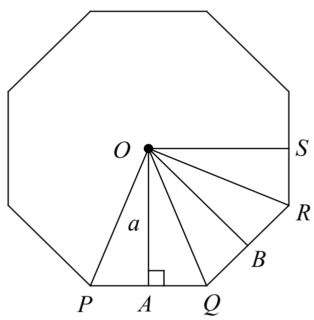
B) 72

C) 76

D) 84

18-3. Regular Polygons

A **regular polygon** is a convex polygon with all sides congruent and all angles congruent.
A polygon is **inscribed in a circle** and the circle is **circumscribed about the polygon** where each vertex of the polygon lies on the circle. The **radius of a regular polygon** is the distance from the center to a vertex of the polygon. A **central angle of a regular polygon** is an angle formed by two radii drawn to consecutive vertices. The **apothem of a regular polygon** is the distance from the center to a side.



Regular Octagon

Center: O
Radius: OP, OQ, OR, $\cdots$
Central Angle: $\angle POQ$, $\angle QOR$, $\cdots$
Interior Angle: $\angle PQR$, $\angle QRS$, $\cdots$
Apothem: OA, OB, $\cdots$ (Denoted with letter a.)

Theorems - Angles and Areas of Regular Polygons

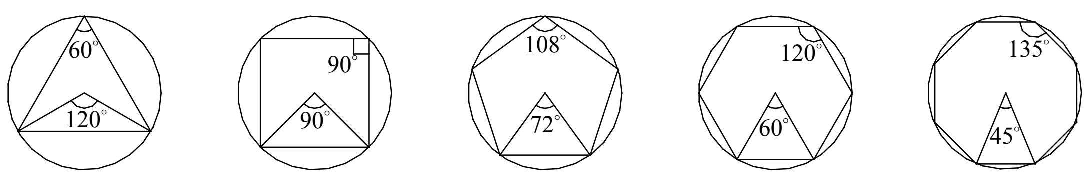
The sum of the measures of the interior angles of an n-sided polygon is $(n-2)180$.

The measure of each interior angle of a regular n-sided polygon is $\dfrac{(n-2)180}{n}$.

The sum of the measures of the exterior angles of any polygon is 360.

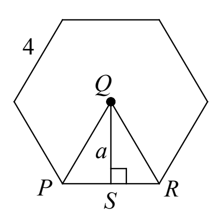
The area of a regular polygon is half the product of the apothem a, and the perimeter p. $A=\dfrac{1}{2}ap$

Regular Polygons Inscribed in Circles



Equilateral Triangle	Square	Regular Pentagon	Regular Hexagon	Regular Octagon
Central angle $=120^\circ$	Central angle $=90^\circ$	Central angle $=72^\circ$	Central angle $=60^\circ$	Central angle $=45^\circ$
Interior angle $=60^\circ$	Interior angle $=90^\circ$	Interior angle $=108^\circ$	Interior angle $=120^\circ$	Interior angle $=135^\circ$

Example 1 □ A regular hexagon with the length of side of 4 is shown at the right.
Find the area of the regular hexagon.



Solution □ $m\angle PQR = 360 \div 6 = 60$

$m\angle PQS = \dfrac{1}{2} m\angle PQR = \dfrac{1}{2}(60) = 30$

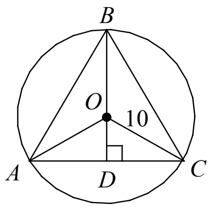
$PS = \dfrac{1}{2} PR = \dfrac{1}{2}(4) = 2$

$a = \sqrt{3} \cdot PS = 2\sqrt{3}$ — 30°-60°-90° triangle ratio is used.

$A = \dfrac{1}{2}ap = \dfrac{1}{2}(2\sqrt{3})(24) = 24\sqrt{3}$ — $A=\dfrac{1}{2}ap$

Exercise - Regular Polygons

Questions 1 - 4 refer to the following information.



The figure above is an equilateral inscribed in a circle with radius 10.

1

What is the measure of $\angle AOC$?

2

What is the length of OD ?

3

What is the length of BD ?

4

What is the area of ΔABC ?

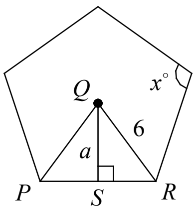
A) $45\sqrt{3}$

B) $50\sqrt{3}$

C) $60\sqrt{3}$

D) $75\sqrt{3}$

Questions 5 - 7 refer to the following information.



The figure above is a regular pentagon whose radius is 6.

5

What is the value of x ?

6

What is the measure of $\angle RQS$?

7

Which of the following equations can be used to find the value of a ?

A) $\sin\angle RQS = \frac{a}{6}$

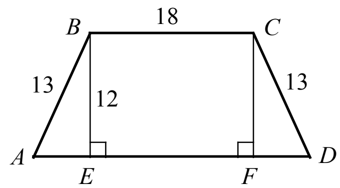
B) $\cos\angle RQS = \frac{a}{6}$

C) $\sin\angle RQS = \frac{6}{a}$

D) $\cos\angle RQS = \frac{6}{a}$

Chapter 18 Practice Test

1



What is the area of the isosceles trapezoid above?

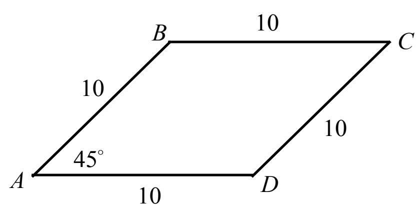
A) 238

B) 252

C) 276

D) 308

2



What is the area of rhombus $ABCD$ above?

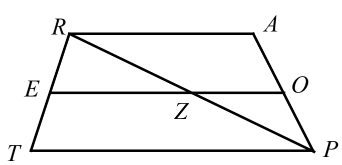
A) $20\sqrt{2}$

B) $25\sqrt{2}$

C) $50\sqrt{2}$

D) $100\sqrt{2}$

3

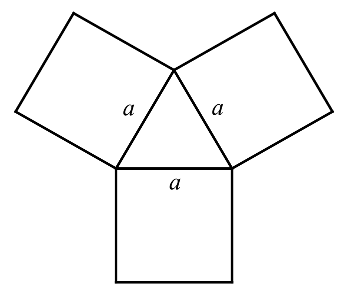


In the figure above, $\overline{EO}$ is the midsegment of trapezoid $TRAP$ and $\overline{RP}$ intersect $\overline{EO}$ at point Z. If $RA = 15$ and $EO = 18$, what is the length of $\overline{EZ}$?

4

A rectangle has a length that is 6 meters more than twice its width. What is the perimeter of the rectangle if the area of the rectangle is 1,620 square meters?

5



The figure above shows an equilateral triangle with sides of length a and three squares with sides of length a. If the area of the equilateral triangle is $25\sqrt{3}$, what is the sum of the areas of the three squares?

A) 210

B) 240

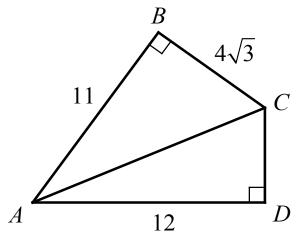
C) 270

D) 300

6

The perimeter of a rectangle is $5x$ and its length is $\frac{3}{2}x$. If the area of the rectangle is 294, what is the value of x ?

7



In the figure above, what is the area of the region $ABCD$?

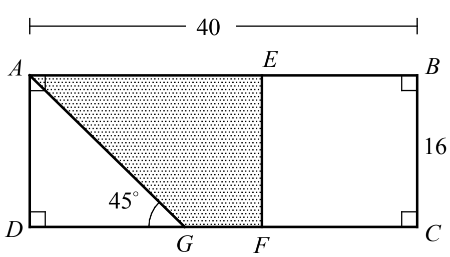
A) $22\sqrt{3}+30$

B) $22\sqrt{3}+36$

C) $22\sqrt{3}+42$

D) $22\sqrt{3}+48$

8



In the figure above, $ABCD$ is a rectangle and $BCFE$ is a square. If $AB=40$, $BC=16$, and $m\angle AGD=45$, what is the area of the shaded region?

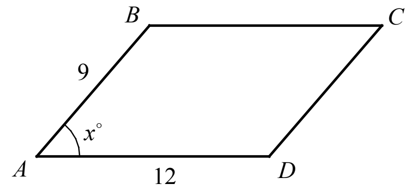
A) 240

B) 248

C) 256

D) 264

9



The figure above shows parallelogram $ABCD$. Which of the following equations represents the area of parallelogram $ABCD$?

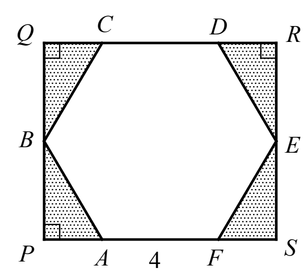
A) $12\cos x^\circ \times 9\sin x^\circ$

B) $12\times 9\tan x^\circ$

C) $12\times 9\cos x^\circ$

D) $12\times 9\sin x^\circ$

10



In the figure above, $ABCDEF$ is a regular hexagon with side lengths of 4. $PQRS$ is a rectangle. What is the area of the shaded region?

A) $8\sqrt{3}$

B) $9\sqrt{3}$

C) $10\sqrt{3}$

D) $12\sqrt{3}$

Answer Key

Section 18-1

1. 4	2. 6	3. 112	4.68	5. 70
6. 24	7. 240			

Section 18-2

1. 25	2. 7.5	3. 27	4. C	5. D

Section 18-3

1. 120	2. 5	3. 15	4. D	5. 108
6. 36	7. B			

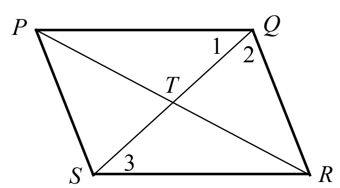
Chapter 18 Practice Test

1. C	2. B	3. 10.5	4. 174	5. D
6. 14	7. A	8. C	9. D	10. A

Answers and Explanations

Section 18-1

1. 4



$PT = \frac{1}{2}PR$	Diagonals of ▱ bisect each other.
$x + 2y = \frac{1}{2}(32) = 16$	Substitution
$ST = TQ$	Diagonals of ▱ bisect each other.
$8x - y = 26$	Substitution
$2(8x - y) = 2(26)$	Multiply each side by 2.
$16x - 2y = 52$	Simplify.

Add $x + 2y = 16$ and $16x - 2y = 52$.

$$\begin{array}{r} 16x - 2y = 52 \\ + \quad x + 2y = 16 \\ \hline 17x = 68 \\ x = 4 \end{array}$$

2. 6

Substitute 4 for x into the equation $x + 2y = 16$.

$4 + 2y = 16$

$2y = 12$

$y = 6$

3. 112

$m\angle 3 = m\angle 1$	If $\overline{PQ} \parallel \overline{RS}$, Alternate Interior $\angle s$ are $\cong$.
$a^2 - 7 = 6a$	Substitution
$a^2 - 6a - 7 = 0$	Make one side 0.
$(a-7)(a+1) = 0$	Factor.
$a = 7$ or $a = -1$	

Discard $a = -1$, because the measure of angles in parallelogram are positive.

$m\angle 1 = 6a = 6(7) = 42$

$m\angle 2 = 10a = 10(7) = 70$

$m\angle PQR = m\angle 1 + m\angle 2$

$= 42 + 70$

$= 112$

4. 68

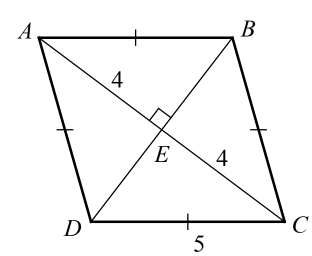
Since $\overline{PQ} \parallel \overline{RS}$, consecutive interior angles are supplementary. Thus, $m\angle PQR + m\angle QRS = 180$.

$112 + m\angle QRS = 180$	$m\angle PQR = 112$
$m\angle QRS = 68$	

5. 70

$m\angle QTR = m\angle PRS + m\angle 3$	Exterior Angle Theorem
$m\angle 3 = m\angle 1 = 42$	
$m\angle PRS = 4a$	Given
$= 4(7) = 28$	$a = 7$
$m\angle QTR = 28 + 42$	Substitution
$= 70$	

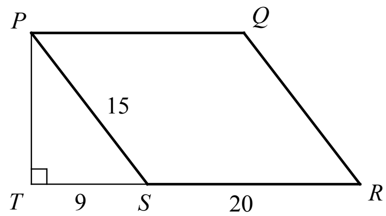
6. 24



$CE^2 + DE^2 = CD^2$	Pythagorean Theorem
$4^2 + DE^2 = 5^2$	
$DE^2 = 9$	
$DE = 3$	

Area of $ABCD = \frac{1}{2}AC \cdot BD = \frac{1}{2}(8)(6) = 24$

7. 240



$PT^2 + ST^2 = PS^2$ Pythagorean Theorem

$PT^2 + 9^2 = 15^2$

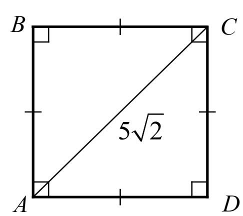
$PT^2 = 15^2 - 9^2 = 144$

$PT = \sqrt{144} = 12$

Area of $PQRS = SR \times PT = 20 \times 12 = 240$.

Section 18-2

1. 25



Let $AD = CD = s$.

$AD^2 + CD^2 = (5\sqrt{2})^2$ Pythagorean Theorem

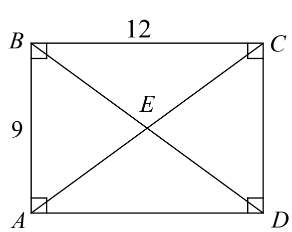
$s^2 + s^2 = 50$

$2s^2 = 50$

$s^2 = 25$

Area of $ABCD = s^2 = 25$.

2. 7.5



$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$AC^2 = 9^2 + 12^2 = 225$ Substitution

$AC = \sqrt{225} = 15$

$AE = \frac{1}{2}AC$ Diagonals of rectangle bisect each other.

$= \frac{1}{2}(15) = 7.5$

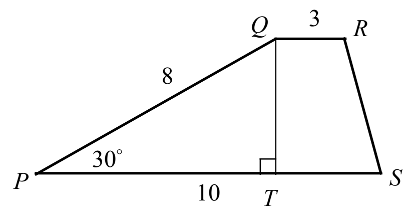
3. 27

Area of rectangle $ABCD = 12 \times 9 = 108$.
In a rectangle, diagonals divide the rectangle into four triangles of equal area. Therefore,

Area of $\Delta CED = \frac{1}{4}$ the area of rectangle $ABCD$

$= \frac{1}{4}(108) = 27$.

4. C



Draw $\overline{QT}$, which is perpendicular to $\overline{PS}$, to make triangle PQT, a 30°-60°-90° triangle.

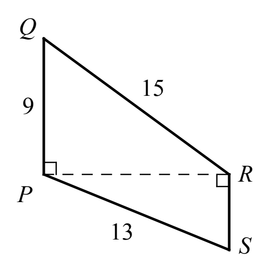
In a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg. Therefore,

$QT = \frac{1}{2}PQ = \frac{1}{2}(8) = 4$.

Area of trapezoid $PQRS = \frac{1}{2}(PS + QR) \cdot QT$

$= \frac{1}{2}(10+3) \cdot 4 = 26$

5. D



$PR^2 + PQ^2 = QR^2$ Pythagorean Theorem

$PR^2 + 9^2 = 15^2$ Substitution

$PR^2 = 15^2 - 9^2 = 144$

$PR = \sqrt{144} = 12$

$12^2 + RS^2 = 13^2$ Pythagorean Theorem

$RS^2 = 13^2 - 12^2 = 25$

$RS = \sqrt{25} = 5$

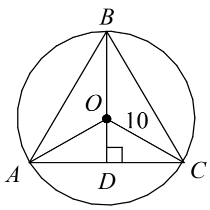
Area of trapezoid $PQRS$

$= \frac{1}{2}(PQ + RS) \cdot PR = \frac{1}{2}(9+5) \cdot 12$

$= 84$

Section 18-3

1. 120



$m\angle AOB = m\angle BOC = m\angle AOC = \frac{1}{3}(360) = 120$

2. 5

$m\angle COD = \frac{1}{2}m\angle AOC = \frac{1}{2}(120) = 60$

Since triangle COD is a 30°-60°-90° triangle, the hypotenuse is twice as long as the shorter leg.

Therefore, $OD = \frac{1}{2}CO = \frac{1}{2}(10) = 5$.

3. 15

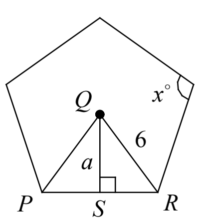
In a circle all radii are equal in measure. Therefore, $AO = BO = CO = 10$.
$BD = BO + OD = 10 + 5 = 15$

4. D

In a 30°-60°-90° triangle, the longer leg is $\sqrt{3}$ times as long as the shorter leg. Therefore,
$CD = \sqrt{3}OD = 5\sqrt{3}$
$AC = 2CD = 10\sqrt{3}$

Area of ΔABC
$= \frac{1}{2}(AC)(BD) = \frac{1}{2}(10\sqrt{3})(15) = 75\sqrt{3}$

5. 108



The measure of each interior angle of a regular n-sided polygon is $\frac{(n-2)180}{n}$. Therefore,

$x = \frac{(5-2)180}{5} = 108$.

6. 36

$m\angle PQR = \frac{360}{5} = 72$

$m\angle RQS = \frac{1}{2}m\angle PQR = \frac{1}{2}(72) = 36$

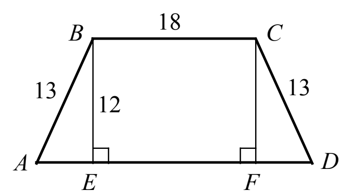
7. B

In triangle RQS, QR is the hypotenuse and QS is adjacent to $\angle RQS$. Therefore the cosine ratio can be used to find the value of a.

$\cos\angle RQS = \frac{\text{adjacent to } \angle RQS}{\text{hypotenuse}} = \frac{a}{6}$

Chapter 18 Practice Test

1. C



$AE^2 + BE^2 = AB^2$ Pythagorean Theorem
$AE^2 + 12^2 = 13^2$

$AE^2 = 13^2 - 12^2 = 25$

$AE = \sqrt{25} = 5$

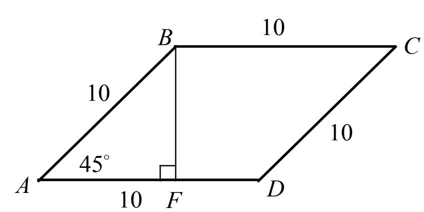
Also $DF = 5$.

$AD = AE + EF + DF = 5 + 18 + 5 = 28$

Area of trapezoid $= \frac{1}{2}(AD + BC)\cdot BF$

$= \frac{1}{2}(28 + 18)\cdot 12 = 276$

2. B



Draw $\overline{BF}$ perpendicular to $\overline{AD}$ to form a 45°-45°-90° triangle.

In a 45°-45°-90° triangle, the hypotenuse is $\sqrt{2}$ times as long as a leg. Therefore, $\sqrt{2}BF = AB$.

$\sqrt{2}BF = 10$ Substitution

$$BF = \frac{10}{\sqrt{2}} = \frac{10\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$$

Area of rhombus $ABCD$

$$= \frac{1}{2}AD\cdot BF = \frac{1}{2}(10)(5\sqrt{2}) = 25\sqrt{2}$$

3. 10.5

The length of the midsegment of a trapezoid is the average of the lengths of the bases. Therefore,

$$EO = \frac{1}{2}(TP + RA).$$

$18 = \frac{1}{2}(TP + 15)$ Substitution

$$2\times 18 = 2\times\frac{1}{2}(TP + 15)$$
$$36 = TP + 15$$
$$21 = TP$$

In ΔTRP, $EZ = \frac{1}{2}TP = \frac{1}{2}(21) = 10.5$.

4. 174

Let $w =$ the width of the rectangle in meters, then $2w + 6 =$ the length of the rectangle in meters.
Area of rectangle = length × width

$$= (2w + 6)\times w = 2w^2 + 6w.$$

Since the area of the rectangle is 1,620 square meters, you can set up the following equation.

$2w^2 + 6w = 1620$

$2w^2 + 6w - 1620 = 0$ Make one side 0.

$2(w^2 + 3w - 810) = 0$ Common factor is 2.

Use the quadratic formula to solve the equation, $w^2 + 3w - 810 = 0$.

$$w = \frac{-b\pm\sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-3\pm\sqrt{3^2 - 4(1)(-810)}}{2(1)}$$
$$= \frac{-3\pm\sqrt{3249}}{2} = \frac{-3\pm 57}{2}$$

Since the width is positive, $w = \frac{-3+57}{2} = 27$.

The length is $2w + 6 = 2(27) + 6 = 60$.

The perimeter of the rectangle is
$2(\text{length} + \text{width}) = 2(60 + 27) = 174$

5. D

Area of an equilateral triangle with side length of $a = \frac{\sqrt{3}}{4}a^2$. Since the area of the equilateral triangle is given as $25\sqrt{3}$, you can set up the following equation.

$$\frac{\sqrt{3}}{4}a^2 = 25\sqrt{3}$$
$$a^2 = 25\sqrt{3}\cdot\frac{4}{\sqrt{3}} = 100$$

The area of each square is a^2, or 100, so the sum of the areas of the three squares is 3×100, or 300.

6. 14

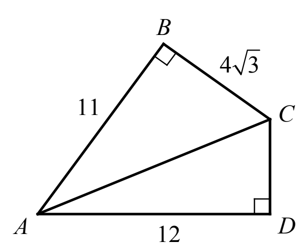
Let $w =$ the width of the rectangle.
The perimeter of the rectangle is given as $5x$.
Perimeter of rectangle = 2(length + width)

$$5x = 2(\frac{3}{2}x + w)$$
$$5x = 3x + 2w$$
$$2x = 2w$$
$$x = w$$

Area of rectangle = length × width = 294

$$\frac{3}{2}x\cdot x = 294$$
$$x^2 = 294\cdot\frac{2}{3} = 196$$
$$x = \sqrt{196} = 14$$

7. A



$AC^2 = AB^2 + BC^2$ Pythagorean Theorem

$AC^2 = 11^2 + (4\sqrt{3})^2$ Substitution

$AC^2 = 121 + 48 = 169$

$AC = \sqrt{169} = 13$

$AC^2 = AD^2 + CD^2$ Pythagorean Theorem

$169 = 12^2 + CD^2$ Substitution

$25 = CD^2$
$5 = CD$

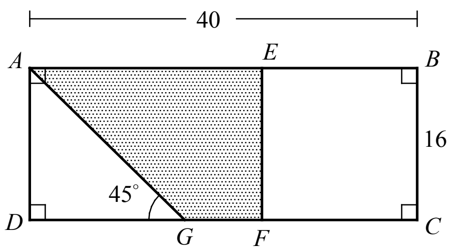
The area of region $ABCD$ is the sum of the area of ΔABC and the area of ΔADC.

Area of the region $ABCD$

$= \frac{1}{2}(11)(4\sqrt{3}) + \frac{1}{2}(12)(5)$

$= 22\sqrt{3} + 30$

8. C



Since $BCFE$ is a square,
$BC = BE = CF = EF = 16$.
$AE = AB - BE$
$= 40 - 16 = 24$

Triangle AGD is a 45°-45°-90° triangle.

In a 45°-45°-90° triangle, the length of the two legs are equal in measure. Therefore,
$AD = DG = 16$.

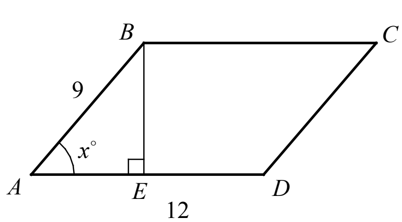
$FG = DC - DG - CF$
$= 40 - 16 - 16 = 8$

Area of the shaded region

$= \frac{1}{2}(AE + FG) \cdot EF$

$= \frac{1}{2}(24 + 8) \cdot 16 = 256$

9. D



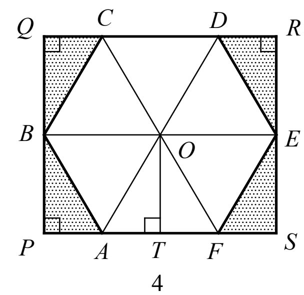
Draw $\overline{BE}$ perpendicular to $\overline{AD}$.

In ΔABE, $\sin x^\circ = \frac{BE}{9}$.

Therefore, $BE = 9\sin x^\circ$.
Area of parallelogram $ABCD$

$= AD \times BE = 12 \times 9 \sin x^\circ$

10. A



Draw the diagonals of a regular hexagon, $\overline{AD}$, $\overline{BE}$, and $\overline{CF}$.
$BE = BO + OE = 8$ and $QR = BE = 8$

Since $ABCDEF$ is a regular hexagon, the diagonals intersect at the center of the hexagon. Let the point of intersection be O. The diagonals divide the hexagon into 6 equilateral triangles with side lengths of 4. Area of each equilateral triangle with side lengths of 4 is $\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$.

Draw $\overline{OT}$ perpendicular to $\overline{PS}$.

Triangle AOT is a 30°-60°-90° triangle.

Therefore, $AT = \frac{1}{2}AO = \frac{1}{2}(4) = 2$ and

$OT = \sqrt{3}AT = 2\sqrt{3}$.

In rectangle $PQRS$, $RS = 2OT = 2(2\sqrt{3}) = 4\sqrt{3}$.
Area of rectangle $PQRS = QR \times RS$
$= 8 \times 4\sqrt{3} = 32\sqrt{3}$.
Area of regular hexagon $ABCDEF$
$= 6 \times$ area of the equilateral triangle
$= 6 \times 4\sqrt{3} = 24\sqrt{3}$
Area of shaded region
$=$ area of rectangle $-$ area of hexagon
$= 32\sqrt{3} - 24\sqrt{3} = 8\sqrt{3}$.