20240207 Test

ANSWER KEY
Diagnostic Test

Section 2

Math Module 1

1. **B**
2. **10**
3. **A**
4. **A**
5. **B**
6. **B**
7. **B**
8. **6**
9. **3**
10. **C**
11. **D**
12. **A**
13. **2, 3, or 4**
14. **D**
15. **A**
16. **25**
17. **C**
18. **D**
19. **A**
20. **25**
21. **B**
22. **C**

Math Module 2

1. **A**
2. **C**
3. **C**
4. **A**
5. **B**
6. **B**
7. **C**
8. **A**
9. **3**
10. **D**
11. **C**
12. **63**
13. **2.74**
14. **D**
15. **A**
16. **A**
17. **D**
18. **C**
19. **64**
20. **B**
21. **D**
22. **0.5 or $\frac{1}{2}$**

Number of Correct Math Questions (Out of 44)	Math Section Score (Out of 800)
0	200
1	220
2	230
3	240
4	250
5	280
6	300
7	320
8	340
9	350
10	360
11	380
12	390
13	400
14	410
15	430
16	440
17	450
18	460
19	480
20	490
21	500
22	510

Number of Correct Math Questions (Out of 44)	Math Section Score (Out of 800)
23	520
24	530
25	540
26	550
27	560
28	570
29	580
30	590
31	600
32	610
33	620
34	630
35	650
36	660
37	680
38	690
39	700
40	720
41	750
42	770
43	790
44	800

Math Module 1

Algebra: 1, 2, 3, 4, 5, 14, 15, 17

Problem Solving and Data Analysis: 6, 7, 12

Advanced Math: 8, 9, 10, 11, 13, 19, 21, 22

Geometry and Trigonometry: 16, 18, 20

Math Module 2

Algebra: 1, 2, 4, 6, 7, 8, 9

Problem Solving and Data Analysis: 10, 11, 13, 16

Advanced Math: 3, 5, 12, 14, 15, 18, 20, 22

Geometry and Trigonometry: 17, 19, 21

Section 2, Module 1: Math

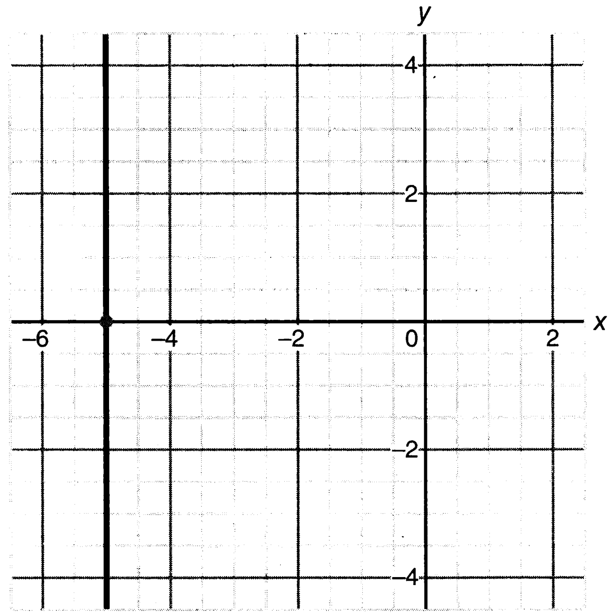
1. **(B)**

$$\frac{6}{7} \times \frac{14}{3} \rightarrow \text{Reduce the 6 and 3} \rightarrow$$
$$\frac{2}{7} \times \frac{14}{1} \rightarrow \text{Reduce the 14 and 7} \rightarrow$$
$$\frac{2}{1} \times \frac{2}{1} = \frac{4}{1} = 4$$

2. **(10)**

$$\frac{2}{3}x - 1 = \frac{1}{6}x + 4 \rightarrow$$
$$\frac{2}{3}x = \frac{1}{6}x + 5 \rightarrow$$
$$\frac{2}{3}x - \frac{1}{6}x = 5 \rightarrow$$
$$\frac{4}{6}x - \frac{1}{6}x = 5 \rightarrow$$
$$\frac{3}{6}x = 5 \rightarrow$$
$$\frac{1}{2}x = 5 \rightarrow x = 10$$

3. **(A)** For a line to be totally vertical, the line must have a constant value for x. $x = -5$ works because no matter what the value of y would be, the value of x will always be -5. Here is what $x = -5$ looks like when graphed:



$y = -4$ represents a horizontal line, and the other options are neither horizontal nor vertical.

4. **(A)**

$$\frac{x}{4} = 6 \rightarrow x = 6 \times 4 \rightarrow x = 24$$

Now that we know $x = 24$, divide 24 by 8 to get the solution:

$$\frac{24}{8} = 3$$

5. **(B)** Put the equation in slope-intercept form: $y = mx + b$ The y-intercept is 0, and the slope is 3—calculate the slope either by visualizing the rise over the run, or by plugging in points to the slope formula. You can use the points (0, 0) and (2, 6):

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{2 - 0} = 3$$

The mass corresponds to the y value and the v corresponds to the x value. So, the equation will then be $m = 3v$.

6. **(B)** The numbers are already in order from least to greatest. Since there is an even number of terms and the two middle terms are different from one another, take the average of the two middle terms, 7 and 9, to find the median of the set:

$$\frac{7 + 9}{2} = 8$$

7. **(B)** Convert from teaspoons, to tablespoons, and then to cups.

$$12 \text{ teaspoons} \times \frac{1 \text{ tablespoon}}{3 \text{ teaspoons}} \times \frac{1 \text{ cup}}{16 \text{ tablespoons}} \rightarrow$$

$$12 \cancel{\text{teaspoons}} \times \frac{1 \cancel{\text{tablespooon}}}{3 \cancel{\text{teaspoons}}} \times \frac{1 \text{ cup}}{16 \cancel{\text{tablespoons}}} = \frac{12}{3 \times 16} = \frac{12}{48} = \frac{1}{4} \text{cup}$$

8. **(6)** Subtract the calories of the cheese and vegetables from the total calories for the salad to find how many calories come from salad dressing:

$$1,100 - 650 = 450$$

Now, divide 450 by 75 to find the number of salad dressing packets used:

$$450 \div 75 = 6$$

So, 6 packets of salad dressing were used.

9. **(3)**

$$\frac{x^2}{6} = \frac{x}{2} \rightarrow \text{Divide both sides by } x \rightarrow$$

$$\frac{x}{6} = \frac{1}{2} \rightarrow x = \frac{6}{2} \rightarrow x = 3$$

10. **(C)** If there is a 30% discount, the new price of the product is 100% $-$ 30% $=$ 70% of the original price. Since the original price is p, take 70% of p. Move the decimal point on 70 to the left two spots, giving you 0.7. Then multiply it by p for the solution: $0.7p$.

11. **(D)** Distance is represented by r in the equation. Double the r to see how the overall force would change:

$$F_g = G\frac{m_1 m_2}{r^2} \rightarrow \text{Double the } r \rightarrow$$

$$F_g = G\frac{m_1 m_2}{(2r)^2} = G\frac{m_1 m_2}{4r^2}$$

So, the force would change such that it is of the original force.

12. **(A)** There are a total of 188 people in the table. Out of these 188, there are 40 male cat owners. So, divide 40 by 188 to get the probability that a randomly selected pet owner would be a male cat owner: $\frac{40}{188}$

13. **(2, 3, or 4)** Any one of these numbers would work as a solution. This is easiest to solve by plugging in some possible numbers. If Connor purchases 2 cheeseburgers for $10, he will need to purchase 3, 4, or 5 hamburgers to be within the $20–$30 range. If Connor purchases 3 cheeseburgers for $15, he will need to purchase 2, 3, or 4 hamburgers to be in the range. If he purchases 4 cheeseburgers for $20, he could purchase 2 hamburgers to be in the range. And if he purchases more than 4 cheeseburgers, he will fall outside the range since he must purchase at least 2 hamburgers.

14. **(D)**

$$12 < -4x + 6y \rightarrow$$

Divide everything by 2 to simplify:

$$6 < -2x + 3y \rightarrow$$

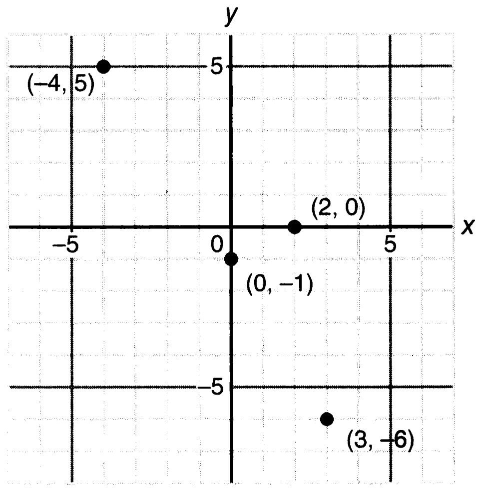
Get y by itself $\rightarrow$

$$6 + 2x < 3y \rightarrow$$

Divide both sides by 3 $\rightarrow$

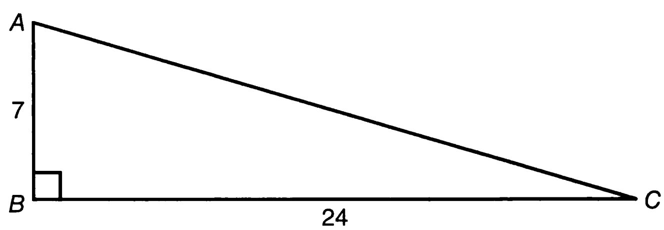
$$2 + \frac{2}{3}x < y$$

15. **(A)**



Given that the line has both a positive slope and a positive y-intercept, the line must be above the x axis for values of x greater than 0. So, choices (B), (C), and (D) will not work since they would fall on or underneath the x-axis for values of x greater than 0. Choice (A) would work because it is already above the x-axis even when it has a negative value for x. A line with the point $(-4, 5)$ could continue with a positive slope to also have a positive y-intercept.

16. **(25)** The triangle can be drawn as follows:



Solve for the unknown side by using the Pythagorean theorem:

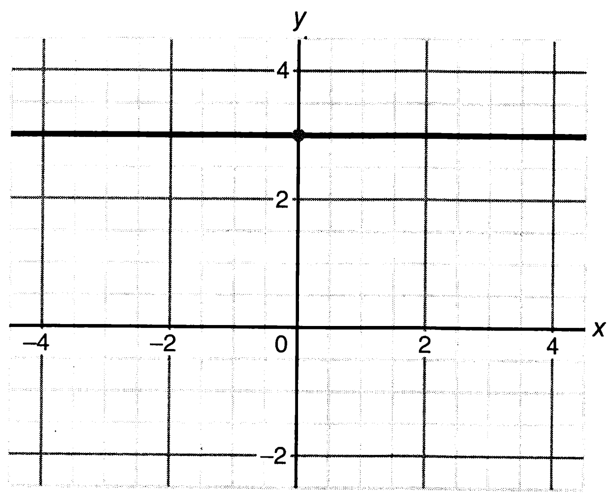
$$a^2 + b^2 = c^2 \rightarrow$$

$$7^2 + 24^2 = 625 = c^2 \rightarrow$$

$$\sqrt{625} = 25 = c$$

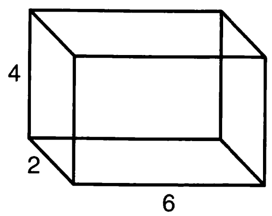
So, the side $AC = 25$. Alternatively, you can recognize that this is a Pythagorean triple that you could have memorized: 7-24-25.

17. **(C)** Plug in a number for k to visualize the situation. Suppose that k is equal to 3. Then, the function would be $f(x) = 3$. The function would look like this:



Condition 1 fits since the function is a line. Condition 2 fits since the function is flat and has a slope of zero. Condition 3 does not fit because the *domain* (corresponds to x values) goes from positive to negative infinity, but the *range* (corresponds to y values) is only 3. So, just conditions 1 and 2 will apply.

18. **(D)** The smallest edge is 2 cm, the next greatest edge will be 4 cm, and the greatest edge will be 6 cm. The prism will look like this:



There are two sides with dimensions 2×4, two sides with dimensions 4×6, and two sides with dimensions 2×6. So, add these surfaces areas together to find the total surface area of the prism:

$$2(2 \times 4) + 2(4 \times 6) + 2(2 \times 6) =$$
$$2(8) + 2(24) + 2(12) =$$
$$16 + 48 + 24 = 88$$

19. **(A)** Use a as the number of aluminum cans and g as the number of glass bottles to create an inequality expressing that the total money made is at least \$100.

$$0.1a + 0.16g \geq 100$$

Then, substitute 600 in for a, since we know that she has collected 600 cans, and solve for g:

$$0.1(600) + 0.16g \geq 100 \rightarrow$$
$$60 + 0.16g \geq 100 \rightarrow$$
$$0.16g \geq 40 \rightarrow$$
$$g \geq \frac{40}{0.16} \rightarrow$$
$$g \geq 250$$

So, the least possible value for the number of bottles would be 250.

20. **(25)** $\angle X$ is an inscribed angle that is across from an intercepted arc that measures 50 degrees. The rule is that an inscribed angle is half the length of the intercepted arc, so take half of 50 to get 25 degrees. You could also estimate this value from the drawing since the figure is drawn to scale.

21. **(B)** The number of minutes spent watering each tree is 5, and the number of minutes water each shrub is 2. So, find the total number of minutes spent watering the sum of the plants by multiplying the number of each plants by the amount of time each plant type is watered:

$$5T + 2S$$

This total must be no more than 30 minutes, meaning it can be less than or equal to 30 minutes. Putting this all together, you have this expression:

$$5T + 2S \leq 30$$

22. **(C)** Use the formula for compound interest:

$A =$ Future Value
$P =$ Initial Value (Principal)
$r =$ Interest Rate Expressed as a Decimal (r is positive if increasing, negative if decreasing)
$t =$ Time
$n =$ Number of Times Interest is Compounded over Time Period t

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

Applying this formula to the problem, v would be the initial value, and t would be the number of years. The constant c corresponds to the part in parentheses: $1 + \frac{r}{n}$. Since the price increases 15% each year, the number of times that it is compounded is just one. So, turn 15% into a decimal to determine what will equal:

$$1 + \frac{r}{n} \rightarrow 1 + \frac{0.15}{1} = 1.15$$

So, the constant c will equal 1.15.

Section 2, Module 2: Math

1. **(A)** Since the slope is defined as the change in y divided by the change in x, and since the mass corresponds to the y value and the v corresponds to the x value, the slope will equal the density of the substance.

2. **(C)**

$$(5x^3 - x + 4) + (6x^3 + x^2 - 3) =$$
$$(5x^3 + 6x^3) + x^2 - x + (4 - 3) =$$
$$11x^3 + x^2 - x + 1$$

3. **(C)** $8^{\left(\frac{1}{3}\right)} \rightarrow \sqrt[3]{8} = 2$

4. **(A)** For the equations to have infinitely many solutions, the two equations should be multiples of each other.

$$2x - 3y = 7$$
$$6x + ky = 21$$

The second equation is three times that of the first equation, since 21 is three times 7 and 6 is three times 2. So, k should be three times -3, which is -9.

5. **(B)** The y-intercept is the point at which the function intersects the y-axis—the value of x at this point must be 0. So, plug 0 in for x to the equation to solve for the y-intercept (keep in mind that a number to the zero power is equal to 1):

$$y = 2^x - 3 \rightarrow y = 2^0 - 3 \rightarrow 1 - 3 = -2$$

6. **(B)** Choice (B) is the only option that will eliminate terms when it is simplified, allowing it to look like $z = x - y$:

$$(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y}) \rightarrow \text{FOIL the expression} \rightarrow$$
$$(\sqrt{x})(\sqrt{x}) + (\sqrt{x})(\sqrt{y}) - (\sqrt{x})(\sqrt{y}) - (\sqrt{y})(\sqrt{y}) =$$
$$x + \cancel{(\sqrt{x})(\sqrt{y})} - \cancel{(\sqrt{x})(\sqrt{y})} - y =$$
$$x - y$$

You could also think about the above calculation as the difference of squares: after *FOILing* the expression, the outer and inner terms cancel, leaving you with just the first and last terms.

7. **(C)** Start by substituting $x + 3$ in for y to the second equation:

$$x(x + 3) = 40$$

Now, solve for x:

$$x(x + 3) = 40 \rightarrow$$
$$x^2 + 3x = 40 \rightarrow$$
$$x^2 + 3x - 40 = 0 \rightarrow$$
$$(x + 8)(x - 5) \rightarrow$$
$$x = -8 \textit{ or } 5$$

Since the question states that both x and y must be positive, use 5 as the value of x. Then, plug 5 in to the first equation to solve for y:

$$x + 3 = y \rightarrow 5 + 3 = 8 = y$$

Now, find the sum of x and y by adding their values together:

$$8 + 5 = 13$$

8. **(A)** The line is horizontal, with every value of x giving the same value of y: 3. So, the equation of the line is $y = 3$.

9. **(3)** Notice a pattern here—if you add the two equations together, you will be able to simplify the expression so that you can solve for $a - b$.

$$\begin{array}{rr} & 2a - 3b = 5 \\ + & \underline{3a - 2b = 10} \\ & 5a - 5b = 15 \end{array}$$

Then, divide the whole equation by 5, and you will have the needed expression:

$$\frac{5a - 5b = 15}{5} \rightarrow a - b = 3$$

Thus, $a - b$ is equal to 3.

10. **(D)** You can eliminate choices (A) and (C) since the mass is going down. And you can eliminate choice (A) because the mass is not decreasing at a steady, linear rate. So, the answer is (D) because the mass is both decreasing and doing so at an exponential rate.

11. **(C)** Sarah traveled 5 miles for 17 minutes. So, convert 5 miles per 17 minutes to miles per hour:

$$\frac{5 \text{ miles}}{17 \text{ \sout{minutes}}} \times \frac{60 \text{ \sout{minutes}}}{1 \text{ hour}} = 17.6 \text{ miles/hour}$$

12. **(63)** Since $(x + y)(x - y) = x^2 - y^2$, substitute 7 and 3 in for these expressions and multiply the product by 3 to arrive at the answer:

$$3x^2 - 3y^2 = 3(x + y)(x - y) = 3(7)(3) = 63$$

13. **(2.74)** Set up an equation that uses x as the number of gallons of Dead Sea water that would be needed to add to the 10 gallons of ocean water to get water with a percent of salt at 10%. The 3.35% is expressed as 0.035 and the 33.7% is expressed as 0.337 since you move the decimal point to the right two spots when calculating percentages. The $10 + x$ represents the total number of gallons.

$$0.035 \times 10 + 0.337x = .1(10 + x) \rightarrow$$
$$0.35 + 0.337x = 1 + 0.1x \rightarrow$$
$$0.337x = .65 + 0.1x \rightarrow$$
$$0.237x = 0.65 \rightarrow$$
$$x = \frac{0.65}{0.237} = 2.74$$

14. **(D)** Looking ahead to the format of the answers, it appears that the quadratic formula will be used to solve for this. Start by putting the equation in quadratic form:

$$4x = 2 - 3x^2 \rightarrow$$
$$3x^2 + 4x - 2 = 0$$

Now, use the quadratic equation to solve for x:

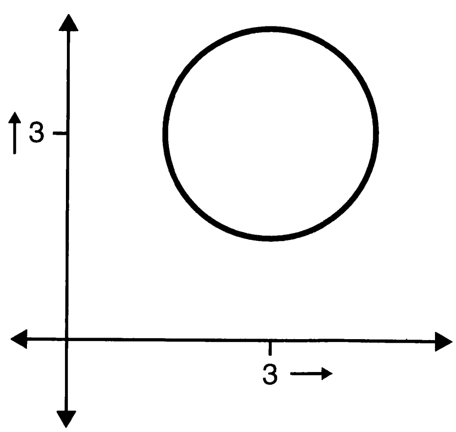
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow$$
$$3x^2 + 4x - 2 = 0 \rightarrow$$
$$a = 3,\ b = 4,\ c = -2$$
$$x = \frac{-4 \pm \sqrt{4^2 - 4(3)(-2)}}{2(3)} \rightarrow$$
$$x = \frac{-4 \pm \sqrt{16 + 24}}{6} \rightarrow$$
$$x = \frac{-4 \pm \sqrt{40}}{6} = \frac{-4 \pm \sqrt{4 \times 10}}{6} \rightarrow$$
$$x = \frac{-4 \pm 2\sqrt{10}}{6} = \frac{-2 \pm \sqrt{10}}{3}$$

Out of the two solutions, $\frac{-2 - \sqrt{10}}{3}$ is the only one given, making (D) correct.

15. **(A)** Factors of the function can be found at the x-intercepts. Look for the x values where $g(x)$ is equal to 0: -2 and 1. Then, see which option would make it so that plugging these two numbers in would result in a value of 0. The only option is choice A. $(x - 1)(x + 2)$

16. **(A)** Since we do not know if the students who participated in the survey were randomly selected, we can only reasonably generalize as to the opinions of those who participated in the survey. So, we can justify the statement that out of the 100 students surveyed, 40 wanted a new library. It is not (B) because we cannot generalize as to the opinions of all university community members based on this non-randomized sample. And it is not (C) or (D) because these options confuse the sample percentage with the number of students.

17. **(D)** The radius of the circle will be 3 since the square root of 9 is 3. So, the values of a and b must each be greater than 3 so that all the coordinates of the circle will be greater than zero—this will place the circle in the first quadrant, as modeled in the graph below:



18. **(C)** Manipulate the equation so that M is isolated.

$$v_e = \sqrt{\frac{2GM}{r}} \rightarrow \text{Square both sides} \rightarrow$$

$$(v_e)^2 = \frac{2GM}{r} \rightarrow \text{Get } M \text{ by itself} \rightarrow$$

$$\frac{(v_e)^2 r}{2G} = M$$

19. **(64)** Write an equation for the original sphere:

$$V = \frac{4}{3}\pi r^3 \rightarrow$$

$$8 = \frac{4}{3}\pi r^3$$

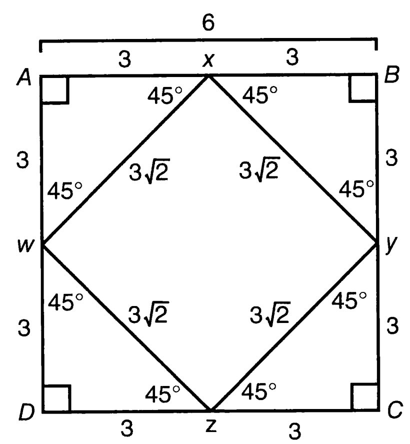
A sphere with a radius of twice this original one would have a radius of $2r$. Calculate the value of this new sphere in terms of r:

$$V = \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r)$$

So, the volume of the new sphere is 8 times the volume of the original sphere: $8 \times 8 = 64$ cubic feet.

20. **(B)** The graph of the function intersects the x-axis at -2, 1, and 4. So, see which equation would have those as zeros, i.e., plugging in these numbers would cause the value of the function to equal zero. The only one that has all three of these numbers as zeros is $y = (x - 4)(x + 2)(x - 1)$

21. **(D)** Since the area of the square is 36 square units, the length of each side will be the square root of 36: 6. Label the drawing to visualize the side lengths and notice that 45-45-90 triangles are inscribed in the larger square, since each side is bisected by a vertex of the inner square.



Add the four sides of square $WXYZ$ to find its perimeter:

$$3\sqrt{2} + 3\sqrt{2} + 3\sqrt{2} + 3\sqrt{2} = 12\sqrt{2}$$

22. **(0.5. or $\frac{1}{2}$)** Solve for the vertex of the parabola, since at that point the projectile will reach its maximum. Use the formula $\frac{-b}{2a}$ to find the time at which it will reach its maximum. The function $h(t) = -16t^2 + 16t + 10$ is already in quadratic form, so you can see the values needed for a and for b. Use for a and 16 for b:

$$\frac{-b}{2a} \rightarrow \frac{-(-16)}{2(16)} = \frac{16}{32} = \frac{1}{2}$$