

SAT Exercise

EXPONENTS, RADICALS, POLYNOMIALS, AND RATIONAL EXPRESSIONS

1

If $\frac{x^c(3x)^2}{9x^3} = x^6$ and $x \neq 0$, what is the value of c ?

2

$$\frac{18x^4 + 27x^3 - 36x^2}{9x^2}$$

If $x \neq 0$, which of the following is equivalent to the given expression?

(A) $2x^2 + 3x - 4$

(B) $2x^2 + 3x - 6$

(C) $2x^4 + 3x^3 - 4x^2$

(D) $2x^6 + 3x^5 - 4x^4$

3

If a and b are positive integer constants and $x^a y^b = -128$, where $x < 0$ and $y < 0$, which of the following must be true?

(A) a is even

(B) a is odd

(C) ab is odd

(D) ab is even

4

If $n^3 = -8$, what is the value of $\frac{(n^2)^3}{\frac{1}{n^2}}$?

5

$$\frac{x^{5r}}{x^{3r-2s}} = x^t$$

If $r + s = 6$ and $x \neq 0$, what is the value of t in the equation shown?

(A) 6

(B) 12

(C) 18

(D) 30

6

$$8 + \frac{\sqrt{2x+29}}{3} = 9$$

For what value of x is this equation true?

(A) -10

(B) -2

(C) 19

(D) No solution

7

$$\begin{aligned} 3x &= x + 14 \\ \sqrt{3z^2 - 11} + 2x &= 22 \end{aligned}$$

If $z > 0$, what is the value of z ?

(A) 1

(B) 3

(C) 5

(D) 8

8

Which of the following expressions is equivalent to $-x^{\frac{1}{4}}$?

(A) $-\frac{1}{4x}$

(B) $-\frac{1}{x^4}$

(C) $-\sqrt[4]{x}$

(D) $\frac{1}{\sqrt[4]{-x}}$

9

When simplified, $8^{\frac{4}{3}}$ is what number?

10

$$\sqrt{3a + 16} - 3 = a - 1$$

In the equation above, if $a \geq 0$, which of the following is a possible value of a ?

(A) 3

(B) 2

(C) 1

(D) 0

11

What is the sum of the polynomials $6a^2 - 17a - 9$ and $-5a^2 + 8a - 2$?

(A) $a^2 - 9a - 11$

(B) $a^2 - 25a - 7$

(C) $11a^2 - 9a - 11$

(D) $11a^2 - 25a - 7$

12

What is the difference when $3x^3 + 7x - 5$ is subtracted from $8x^2 + 4x + 10$?

(A) $5x^2 - 3x + 15$

(B) $-3x^3 - 3x + 5$

(C) $3x^3 - 8x^2 + 3x - 15$

(D) $-3x^3 + 8x^2 - 3x + 15$

13

If $A = 4x^2 + 7x - 1$ and $B = -x^2 - 5x + 3$, then what is $\frac{3}{2}A - 2B$?

(A) $4x^2 + \frac{31}{2}x - \frac{9}{2}$

(B) $4x^2 + \frac{41}{2}x - \frac{15}{2}$

(C) $8x^2 + \frac{31}{2}x - \frac{9}{2}$

(D) $8x^2 + \frac{41}{2}x - \frac{15}{2}$

14

If $x^3 - 9x = 9 - x^2$, which of the following CANNOT be the value of x ?

(A) -3

(B) -1

(C) 1

(D) 3

15

$$(2x^2 + 3x - 4)(3x + 2) = 6x^3 + ax^2 - 6x - 8$$

In the given equation, a is a constant. If the equation is true for all values of x , what is the value of a ?

(A) 4

(B) 9

(C) 13

(D) 16

16

Which of the following is equivalent to

$$\frac{2a^2 - 5a - 1}{a - 3}?$$

- (A) $2a - 2$
- (B) $2a + 1 - \frac{2}{a - 3}$
- (C) $2a + \frac{2}{a - 3}$
- (D) $2a + 1 + \frac{2}{a - 3}$

17

$$\frac{6x^2 + 19x + 10}{2x + 5}$$

If $ax + b$ represents the simplified form of the expression shown, then what is the value of $a + b$?

- (A) 2
- (B) 3
- (C) 5
- (D) 6

18

Which of the following is equivalent to $\frac{4x^2 - 6x}{2x + 2}$?

- (A) $2x - \frac{10}{2x + 2}$
- (B) $2x - 5 + \frac{10}{2x + 2}$
- (C) $2x - 3$
- (D) $2x + 5 - \frac{10}{2x + 2}$

19

The equation $\frac{36x^2 + 16x - 21}{tx - 4} = -9x + 5 - \frac{1}{tx - 4}$ is true for all values of x for which $x \neq \frac{4}{t}$, where t is a constant. What is the value of t ?

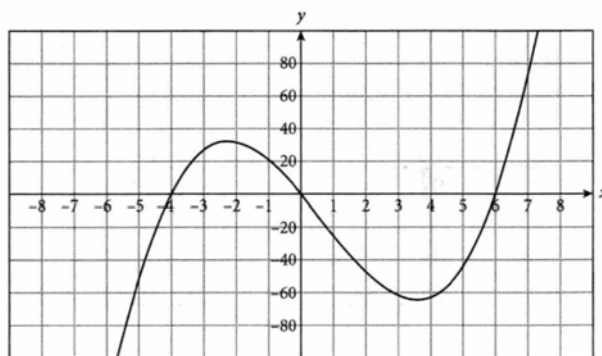
- (A) -20
- (B) -4
- (C) 4
- (D) 12

20

If the polynomial $f(x)$ is evenly divisible by $x - 5$ and the polynomial $g(x) = f(x) + 4$, what is the value of $g(5)$?

- (A) -4
- (B) 0
- (C) 4
- (D) 9

21



Which of the following could be the equation of the function in the graph?

- (A) $f(x) = x^2(x + 4)(x - 6)$
- (B) $f(x) = x(x + 4)(x - 6)$
- (C) $f(x) = x^2(x - 4)(x + 6)$
- (D) $f(x) = x(x - 4)(x + 6)$

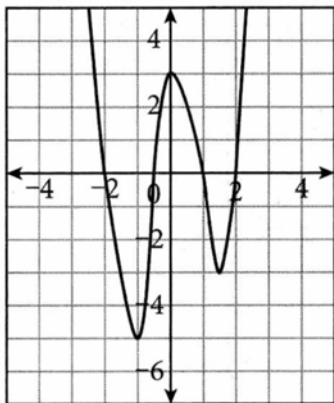
22

x	$h(x)$
-3	6
-1	0
0	-5
2	-8

The function h is defined by a polynomial. The table shown gives some of the values of x and $h(x)$. Which of the following must be a factor of $h(x)$?

- (A) $x - 8$
- (B) $x - 1$
- (C) $x + 1$
- (D) $x + 5$

23



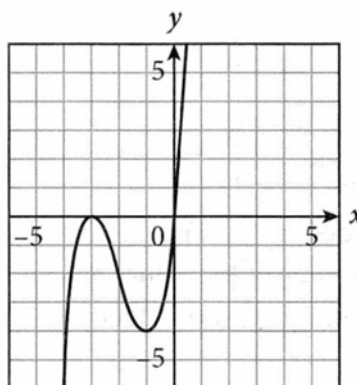
The graph of the function $a(x)$ is shown. If $b(x) = \frac{1}{x}$, which of the following is a true statement about $b(a(x))$?

- (A) $b(a(x))$ is defined for all real numbers.
- (B) $b(a(x))$ is undefined for exactly one real value of x .
- (C) $b(a(x))$ is undefined for at least four real values of x .
- (D) $b(a(x))$ is undefined for all real numbers.

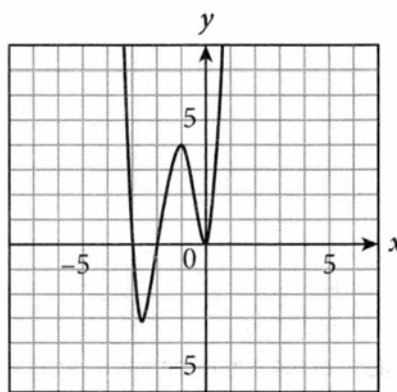
24

If function f has exactly two distinct real zeros, which of the following graphs could be the complete graph of $f(x)$?

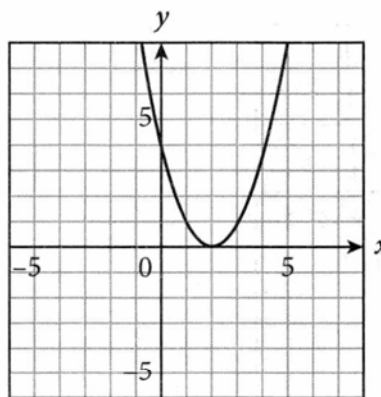
(A)



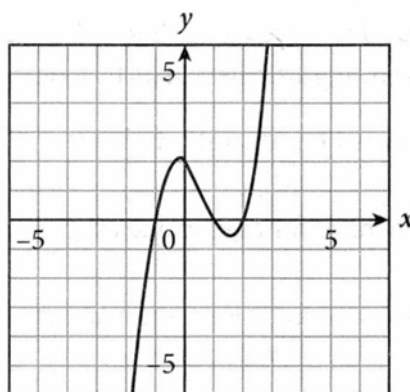
(B)



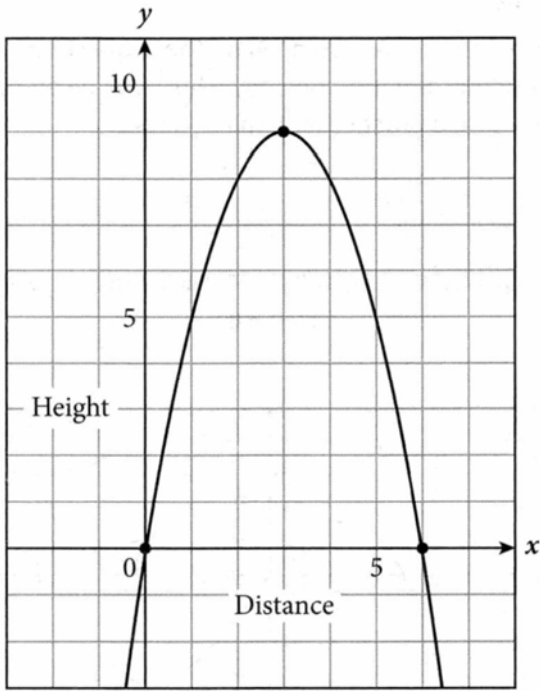
(C)



(D)



25



The graph of $f(x) = -(x - 3)^2 + 9$ approximates the trajectory of a water balloon shot from a cannon at ground level. In terms of the trajectory, what information is represented by a root of this function?

- (A) The maximum height achieved by the balloon
- (B) The total horizontal distance traveled by the balloon
- (C) The maximum speed of the balloon
- (D) The initial acceleration of the balloon

26

In determining the winner of a speech-writing competition, a panel of judges eliminates one-quarter of the remaining applicants per day of deliberations. If 128 students entered the competition, how many applicants have been eliminated by the end of the third day of deliberations?

27

A health club's membership has increased at a rate of 16 percent per year for the past four years. The club currently has 42 members. If this trend continues, how many years will it take for the club's membership to exceed 100 members?

- (A) 4 years
- (B) 5 years
- (C) 6 years
- (D) 7 years

28

^{14}C (an isotope of carbon) has a half-life of 5,600 years, which means half of the ^{14}C in the remains of an organism will decay in that time period. If a sample of a petrified tree contains 6.25 percent of its original ^{14}C , how long ago did the tree die?

- (A) 22,400 years
- (B) 28,000 years
- (C) 35,000 years
- (D) 89,600 years

29

Penelope receives the same amount of money each month for her allowance. Each month she spends half of her allowance and puts the rest in a piggy bank. On her 8th birthday, the piggy bank contains \$40. If the piggy bank contains \$244 after 2 years, what is her monthly allowance in dollars?

30

Account X earns a monthly interest equal to 2 percent of the original investment, while account Y earns a monthly interest equal to 2 percent of the current value of the account. If \$500 is invested into each account, what is the positive difference between the value of account X and account Y after three years? (Round your answer to the nearest dollar.)

31

Given the equation $\frac{9}{x} = \frac{2}{k+2}$ and the constraints $x \neq 0$ and $k \neq -2$, what is x in terms of k ?

(A) $x = 2k + 4$

(B) $x = 2k + 12$

(C) $x = 2k - \frac{1}{4}$

(D) $x = \frac{1}{4}k + 12$

32

$$\frac{3a+9}{(a-3)^2} + \frac{-9}{3a-9}$$

In the given expression, $(a-3)^2 = 6$. What is the value of the expression?

33

If $a > 6$, which of the following is equivalent to

$$\frac{\frac{2}{a}}{\frac{1}{a-2} + \frac{1}{a-6}}?$$

(A) $2a^2 - 16a + 24$

(B) $a(2a - 8)$

(C) $\frac{a^2 - 8a + 12}{a^2 - 4a}$

(D) $\frac{2a - 8}{a^2 - 8a + 12}$

34

If $\frac{x^3 - 3x^2}{x - 3} = 9$, what is the value of x ?

(A) -3

(B) 0

(C) 3

(D) 9

35

If $\frac{16}{7x+4} + A$ is equivalent to $\frac{49x^2}{7x+4}$, what is A in terms of x ?

(A) $7x + 4$

(B) $7x - 4$

(C) $49x^2$

(D) $49x^2 + 4$

36

If $\frac{1-2c}{3c} - \frac{c-8}{12} = 0$ and $c < 0$, what is the value of c ?

(A) -4

(B) -2

(C) 1

(D) 2

SAT Exercise Answers

EXPONENTS, RADICALS, POLYNOMIALS, AND RATIONAL EXPRESSIONS

1. 7

Difficulty: Easy**Category:** Advanced Math

Getting to the Answer: Simplify the fraction on the left side of the equation using exponent rules. Start by distributing the power of 2 outside the parentheses to both the 3 and the x inside:

$$\begin{aligned}\frac{x^c(3x)^2}{9x^3} &= x^6 \\ \frac{x^c(3)^2(x^2)}{9x^3} &= x^6 \\ \frac{9(x^c)(x^2)}{9x^3} &= x^6\end{aligned}$$

Now, cancel the 9s in the numerator and denominator, and add the exponents of the two x terms, since it's multiplication of the same base to different exponents:

$$\frac{\cancel{9}(x^{c+2})}{\cancel{9}x^3} = x^6$$

There's now an x in both the numerator and the denominator raised to different exponents, so subtract them:

$$\begin{aligned}x^{(c+2)-3} &= x^6 \\ x^{c-1} &= x^6\end{aligned}$$

Since, according to the question, $x \neq 0$, this means that $c - 1 = 6$, or $c = 7$. Enter **7**.

2. A

Difficulty: Easy**Category:** Advanced Math

Getting to the Answer: Find the greatest common factor (GCF) of both the numerator and the denominator, which in this question happens to be the denominator. Factor out the GCF, $9x^2$, from the numerator and denominator and then cancel what you can:

$$\begin{aligned}\frac{18x^4 + 27x^3 - 36x^2}{9x^2} &= \frac{\cancel{9}x^2(2x^2 + 3x - 4)}{\cancel{9}x^2} \\ &= 2x^2 + 3x - 4\end{aligned}$$

This matches **(A)**. As an alternate method, you could split the expression up and reduce each term, one at a time:

$$\begin{aligned}\frac{18x^4 + 27x^3 - 36x^2}{9x^2} &= \frac{18x^4}{9x^2} + \frac{27x^3}{9x^2} - \frac{36x^2}{9x^2} \\ &= 2x^2 + 3x - 4\end{aligned}$$

3. D

Difficulty: Hard**Category:** Advanced Math

Getting to the Answer: If the product of x^a and y^b is negative, then either x^a is positive and y^b is negative, or vice versa. A negative number raised to an even exponent is positive, and a negative number raised to an odd exponent is negative. Since x and y are both negative, then either a is even and b odd, or vice versa.

Now, evaluate the choices to see what must be true. (A) says that a is even; this is possibly true, but it's also possible that b is even and a is odd. Thus, (A) can be eliminated. (B) can be eliminated for the same reason. (C) says that ab is odd. The only way for the product of two integers to be odd is if both of the integers are odd; since one of either a or b must be even, ab must be even. Eliminate (C); **(D)** is correct.

If you are ever unsure of even and odd rules, you can pick numbers to test them out: Odd \times Even = Even; $3 \times 4 = 12$.

4. 256

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: If $n^3 = -8$, then $n = -2$. Plug -2 in for n and simplify the given expression via exponent rules:

$$\frac{((-2)^2)^3}{(-2)^2} = \frac{4^3}{4} = 4^3 \times 4 = 4^4 = 256$$

Enter **256**.

5. B

Difficulty: Hard**Category:** Advanced Math

Getting to the Answer: Because the bases are the same, to simplify the fraction on the left side of the equation just subtract the powers and combine:

$$\begin{aligned}\frac{x^{5r}}{x^{3r-2s}} &= x^{5r-(3r-2s)} \\ &= x^{5r-3r+2s} \\ &= x^{2r+2s}\end{aligned}$$

Note that in the expression $2r + 2s$, it is possible to factor out a 2. Thus, $x^{2r+2s} = x^{2(r+s)}$. The question indicates that $r + s = 6$, so $x^{2(r+s)} = x^{2(6)} = x^{12}$. This is equal to x^t , so $t = 12$. The answer is **(B)**.

6. A

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: Solve equations containing radical expressions the same way you solve any other equation: isolate the variable using inverse operations. Start by subtracting 8 from both sides of the equation and then multiply by 3. Then, square both sides to remove the radical:

$$\begin{aligned} 8 + \frac{\sqrt{2x+29}}{3} &= 9 \\ \frac{\sqrt{2x+29}}{3} &= 1 \\ \sqrt{2x+29} &= 3 \\ 2x+29 &= 9 \end{aligned}$$

Now you have a simple linear equation that you can solve using more inverse operations: subtract 29 and divide by 2 to find that $x = -10$. Be careful—just because the equation started with a radical and the answer is negative, it does not follow that *No solution* is the correct answer. If you plug -10 into the expression under the radical, the result is a positive number, which means -10 is a perfectly valid solution. Therefore, **(A)** is correct.

7. C

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: Subtract x from both sides of the first equation to get $2x = 14$. You could solve for x , but since $2x$ appears in the second equation, plug it in to get $\sqrt{3z^2 - 11} + 14 = 22$. Thus, $\sqrt{3z^2 - 11} = 8$. Square both sides of this equation and solve for z :

$$\begin{aligned} 3z^2 - 11 &= 64 \\ 3z^2 &= 75 \\ z^2 &= 25 \\ z &= \pm 5 \end{aligned}$$

8. C

Difficulty: Easy**Category:** Advanced Math

Getting to the Answer: Follow the standard order of operations—deal with the exponent first and then attach the negative sign (because a negative in front of an expression means multiplication by -1). The variable x is being raised to the $\frac{1}{4}$ power, so rewrite the term as a radical expression with 4 as the degree of the root and 1 as the power to which the radicand, x , is being raised:

$$x^{\frac{1}{4}} = \sqrt[4]{x^1} = \sqrt[4]{x}$$

Now attach the negative to arrive at the correct answer, $-\sqrt[4]{x}$, which is **(C)**.

9. 16

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: While this could be solved for with a calculator, learning exponent rules may save you time in the long run. The following calculation could also be done quickly with mental math after some practice. Rewrite the exponent in a way that makes it easier to evaluate: use exponent rules to rewrite $\frac{4}{3}$ as a unit fraction raised to a power. Then write the expression in radical form and simplify:

$$\begin{aligned} 8^{\frac{4}{3}} &= \left(8^{\frac{1}{3}}\right)^4 \\ &= (\sqrt[3]{8})^4 \\ &= 2^4 \\ &= 2 \times 2 \times 2 \times 2 \\ &= 16 \end{aligned}$$

Enter 16.

10. A

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: This is a question that lends itself to backsolving. Each answer is easily plugged into the equation and checked:

Choice (A):

$$\begin{aligned} \sqrt{3a+16} - 3 &= a - 1 \\ \sqrt{3(3)+16} - 3 &= 3 - 1 \\ \sqrt{25} - 3 &= 2 \\ 5 - 3 &= 2 \end{aligned}$$

Choice **(A)** is correct. For the record:

Choice (B) would yield the false statement $\sqrt{22} - 3 = 1$ and would be eliminated.

Choice (C) would yield the false statement $\sqrt{19} - 3 = 0$ and would be eliminated.

Choice (D) would yield the false statement $4 - 3 = -1$ and would be eliminated.

You could also solve this with quadratics (which is covered in a later chapter). Start by isolating the radical on the left side of the equation by adding 3 to both sides to get $\sqrt{3a+16} = a+2$. Now you can square both sides to get rid of the radical: $3a+16 = (a+2)^2 = a^2+4a+4$. Since the right side of this equation is a quadratic, set it equal to 0 in order to determine the solutions:

$0 = a^2 + 4a + 4 - (3a + 16) = a^2 + a - 12$. Next, factor the quadratic using reverse FOIL. The two factors of -12 that add up to 1 are -3 and 4 , so $(a - 3)(a + 4) = 0$. Thus, a can be either 3 or -4 , but the question says $a \geq 0$, so the only permissible value is 3 . **(A)** is correct.

11. A**Difficulty:** Easy**Category:** Advanced Math

Getting to the Answer: Add polynomial expressions by combining like terms. Be careful of the signs of each term. It may help to write the sum vertically, lining up the like terms:

$$\begin{array}{r} 6a^2 - 17a - 9 \\ + (-5a^2 + 8a - 2) \\ \hline a^2 - 9a - 11 \end{array}$$

The correct choice is **(A)**.

12. D**Difficulty:** Medium**Category:** Advanced Math

Getting to the Answer: First, write the question as a subtraction problem. Pay careful attention to which expression is being subtracted so that you distribute the negative sign correctly, and make sure you only subtract terms with the same base and exponent.

$$8x^2 + 4x + 10 - (3x^3 + 7x - 5) = -3x^3 + 8x^2 - 3x + 15$$

This expression matches **(D)**.

13. D**Difficulty:** Medium**Category:** Advanced Math

Getting to the Answer: Multiply each term in the first expression by $\frac{3}{2}$ and each term in the second expression by 2 . Then, subtract the two polynomials by writing them vertically and combining like terms. You'll have to find a common denominator to combine the x -coefficients and to combine the constant terms:

$$\begin{array}{r} \frac{3}{2}A = \frac{3}{2}(4x^2 + 7x - 1) = 6x^2 + \frac{21}{2}x - \frac{3}{2} \\ 2B = 2(-x^2 - 5x + 3) = -2x^2 - 10x + 6 \\ \hline 6x^2 + \frac{21}{2}x - \frac{3}{2} \\ - (-2x^2 - \frac{20}{2}x + \frac{12}{2}) \\ \hline 8x^2 + \frac{41}{2}x - \frac{15}{2} \end{array}$$

This means **(D)** is correct. Notice that if you are simplifying the expression from left to right, after you find the x^2 -coefficient, you can eliminate (A) and (B). After you find the x -coefficient, you can eliminate (C) and stop your work.

14. C**Difficulty:** Hard**Category:** Advanced Math

Getting to the Answer: In order to solve the equation, move all the terms to one side of the equation to set them equal to 0 , then factor the expression. Thus, the given equation becomes $x^3 + x^2 - 9x - 9 = 0$. Think of this as two pairs of terms, $(x^3 + x^2)$ and $(-9x - 9)$. The first pair of terms share a common factor of x^2 , so they can be written as $x^2(x + 1)$. The second pair share the common factor of -9 , so they are equivalent to $-9(x + 1)$. So, the equation becomes $x^2(x + 1) - 9(x + 1) = 0$. Now, factor out the $(x + 1)$ term: $(x^2 - 9)(x + 1) = 0$.

In order for the product of two terms to be 0 , either one or both must be 0 . If $x^2 - 9 = 0$, then $x^2 = 9$ and $x = \pm 3$. Eliminate (A) and (D). If $x + 1 = 0$, then $x = -1$. Eliminate (B), so **(C)** is correct. You could also answer the question using Backsolving by plugging in each answer choice until you found the value for x that did *not* satisfy the equation.

15. C**Difficulty:** Medium**Category:** Advanced Math

Strategic Advice: To multiply two polynomials, multiply each term in the first factor by each term in the second factor, then combine like terms.

Getting to the Answer: Multiply each part of the trinomial expression by each part of the binomial one piece at a time and then combine like terms:

$$\begin{aligned} &(2x^2 + 3x - 4)(3x + 2) \\ &= 2x^2(3x + 2) + 3x(3x + 2) - 4(3x + 2) \\ &= 6x^3 + 4x^2 + 9x^2 + 6x - 12x - 8 \\ &= 6x^3 + 13x^2 - 6x - 8 \end{aligned}$$

Because a represents the coefficient of x^2 , $a = 13$. Hence, **(C)** is correct.

16. D

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2a + 1 \\ a - 3 \overline{) 2a^2 - 5a - 1} \\ \underline{-(2a^2 - 6a)} \\ a - 1 \\ \underline{-(a - 3)} \\ 2 \end{array}$$

The quotient is $2a + 1$ and the remainder is 2, which will be divided by the divisor in the final answer: $2a + 1 + \frac{2}{a - 3}$. Thus, (D) is correct.

17. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: A fraction is the same as division, so you can use polynomial long division to simplify the expression:

$$\begin{array}{r} 3x + 2 \\ 2x + 5 \overline{) 6x^2 + 19x + 10} \\ \underline{-(6x^2 + 15x)} \\ 4x + 10 \\ \underline{-(4x + 10)} \\ 0 \end{array}$$

The simplified expression is $3x + 2$, so $a = 3$ and $b = 2$, and $a + b = 3 + 2 = 5$, which is (C). As an alternate approach, you could factor the numerator of the expression and cancel common factors:

$$\frac{6x^2 + 19x + 10}{2x + 5} = \frac{(2x + 5)(3x + 2)}{(2x + 5)} = 3x + 2$$

18. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: Use polynomial long division to simplify the expression:

$$\begin{array}{r} 2x - 5 \\ 2x + 2 \overline{) 4x^2 - 6x} \\ \underline{-(4x^2 + 4x)} \\ -10x \\ \underline{-(-10x - 10)} \\ +10 \end{array}$$

The quotient is $2x - 5$ and the remainder is 10. Put the remainder over the divisor and add this to the quotient: $2x - 5 + \frac{10}{2x + 2}$ (B) is correct.

19. B

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: The question provides the quotient of $-9x + 5$ of a division problem and asks you to find the coefficient of the first term of the divisor $tx - 4$. Set this up in polynomial long division form to better understand the relationship between t and the other terms:

$$tx - 4 \overline{) -9x + 5}$$

Viewed this way, it becomes apparent that $36x^2 \div tx = -9x$. Multiplying both sides by tx gives you $tx(-9x) = 36x^2$; therefore, $t(-9) = 36$, so $t = -4$. (B) is correct.

20. C

Difficulty: Hard

Category: Advanced Math

Getting to the Answer: Because $f(x)$ is divisible by $x - 5$, the value $x - 5$ must be a factor of $f(x)$. Therefore, you can define $f(x)$ as $(x - 5)(n)$, where n is some unknown polynomial. Since $g(x)$ is $f(x) + 4$, you can say that $g(x)$ must be $(x - 5)(n) + 4$.

Thus, $g(5)$ will be $(5 - 5)(n) + 4 = 0(n) + 4 = 0 + 4 = 4$. Therefore, (C) is correct.

21. B

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: The solutions, or x -intercepts, of a polynomial are the factors of that polynomial. This polynomial has x -intercepts of -4 , 0 , and 6 . The factors that generate those solutions are $(x + 4)$, x , and $(x - 6)$. Eliminate (C) and (D) because they do not include those three factors. Because the graph crosses the x -axis at each x -intercept (rather than merely touching the x -axis), none of the factors can be raised to an even exponent. Therefore, eliminate (A) because of the x^2 term. (B) is correct.

22. C

Difficulty: Medium

Category: Advanced Math

Getting to the Answer: To find the solutions to a polynomial function, factor the polynomial and set each factor equal to 0. The solutions of a function are the x -intercepts, so $h(x)$ or the y -coordinate of the solution must equal 0. From the chart, the only point with $h(x) = 0$ is at $x = -1$. If $x = -1$, the factor that generates that solution is $x + 1 = 0$ because $(-1) + 1 = 0$. (C) is correct.

23. C**Difficulty:** Hard**Category:** Advanced Math

Getting to the Answer: Translate the notation: $b(a(x))$ means b of $a(x)$. This tells you to use $a(x)$ as the input for $b(x)$. You can rewrite this as $\frac{1}{a(x)}$, which is the reciprocal of $a(x)$. This new function will be undefined anywhere that $a(x) = 0$ because a denominator of 0 is not permitted. Looking at the graph, you can see that $a(x)$ crosses the x -axis four times, at which point the value of $a(x)$ is 0. Since division by 0 is undefined, $b(a(x))$ will be undefined for at least these four points, so **(C)** is correct.

24. A**Difficulty:** Easy**Category:** Advanced Math

Getting to the Answer: The phrase “exactly 2 distinct real zeros” means that the graph must have exactly two different x -intercepts on the graph. An x -intercept is indicated any time that the graph either crosses or touches the x -axis. (B) and (D) have three distinct zeros, and (C) has two zeros, but because the graph only touches the x -axis, they are the same, not distinct. The only graph with exactly two distinct zeros is **(A)**.

25. B**Difficulty:** Easy**Category:** Advanced Math

Getting to the Answer: The keyword “root” in the question stem means that you should examine the places at which the graph intersects the x -axis. Thus, this graph has roots at $(0, 0)$ and $(6, 0)$. The x -axis, according to the graph, represents the horizontal distance traveled by the balloon. When $x = 0$, the distance the water balloon has traveled is 0, which is the balloon’s starting position. The initial location of the balloon is not an answer choice, so the correct answer must be what the other root represents. When $x = 6$, the balloon’s height is 0, which is the end point of the balloon’s trajectory. This value, 6, is a root that represents the total horizontal distance traveled. **(B)** is correct.

26. 74**Difficulty:** Medium**Category:** Advanced Math

Strategic Advice: The goal is to find the number of applicants *eliminated* after three days, not the number remaining.

Getting to the Answer: The question describes the decay as the result of removing a certain fraction of the remaining applicants each day. The situation involves repeated division, so this is an example of exponential decay. You could use the exponential decay formula for a given rate, but it may be more straightforward to determine how many applicants are eliminated each day and tally them up.

After the first day, the judges eliminate one-fourth of 128, or 32, applicants. This leaves $128 - 32 = 96$ applicants. On the second day, one-fourth of 96, or 24, applicants are eliminated, leaving $96 - 24 = 72$. Finally, on the third day, one-fourth are eliminated again; one-fourth of 72 is 18, so there are $72 - 18 = 54$ applicants remaining. If 54 applicants remain, then $128 - 54 = 74$ applicants have been eliminated. Enter **74**.

27. C**Difficulty:** Medium**Category:** Advanced Math

Strategic Advice: This question gives you a percent increase per year, so use the exponential growth equation to solve for the number of years.

Getting to the Answer: Use the formula for exponential growth and plug in the values from the question. The rate is 16%, which as a decimal is 0.16. The rate will remain positive because the question asks about increase, or growth; therefore, $r = 0.16$. The current number of members is 42, so this will be $f(0)$. The goal is at least 100 members, so that will be the output, or $f(t)$. Put it all together:

$$\begin{aligned} f(t) &= f(0)(1 + r)^t \\ 100 &= 42(1 + 0.16)^t \\ 100 &= 42(1.16)^t \end{aligned}$$

At this point, backsolving is the best approach. Plug in the number of years for t . Because the answer choices are in ascending order, try one of the middle options first. You might be able to eliminate more than one choice at a time. Choice (B) is $t = 5$:

$$42(1.16)^5 \approx 88$$

Since (B) is too small, (A) must be as well. Eliminate them both. Unfortunately, 88 is not close enough to 100 to be certain that **(C)** is the correct answer, so test it:

$$42(1.16)^6 \approx 102$$

Six years is enough to put the club over 100 members. **(C)** is correct.

28. A**Difficulty:** Hard**Category:** Advanced Math

Strategic Advice: The term “half-life” signals exponential decay because it implies repeated division by 2. Using the exponential decay formula here could be complicated. Instead, you can use the percentage given in the question, along with the picking numbers strategy, to figure out how many half-lives have elapsed.

Getting to the Answer: Instead of providing an actual amount of ^{14}C , this question tells you what percent is left. For questions involving percentages of unknown values, it is often a good idea to pick 100. So, assume that the amount of ^{14}C in the sample when the tree died is 100. (Fortunately, there is no need to worry about the units here.) After one half-life, the amount of ^{14}C is halved to 50. A second half-life leaves 25, a third leaves 12.5, and a fourth leaves 6.25, which is 6.25% of 100. So four half-lives have elapsed. Since each half-life is 5,600 years, the tree died $4 \times 5,600$ or 22,400 years ago. Choice **(A)** is correct.

29. 17**Difficulty:** Medium**Category:** Advanced Math

Strategic Advice: The question describes a situation with linear growth since Penelope is adding the same amount of money to her piggy bank each month. Note: the question is asking for her monthly allowance, but she puts in only half that amount each month.

Getting to the Answer: Use the linear growth equation $y = mx + b$. The question gives you the starting amount b (\$40), the final amount y (\$244), and the amount of time x (2 years, which is 24 months). Plug these values into the equation and solve for m , which is the slope, or the rate of change—or in this case, how much Penelope puts in her piggy bank each month:

$$\begin{aligned}y &= mx + b \\244 &= m(24) + 40 \\24m &= 204 \\m &= 8.5\end{aligned}$$

Remember that what she puts in the piggy bank is only half of her allowance, so her total monthly allowance is twice \$8.50. Enter **17**.

30. 160**Difficulty:** Hard**Category:** Advanced Math

Strategic Advice: This question describes both types of growth. Account X adds a percentage of the original amount, which never changes, so the same amount of money is added each month. Account X grows linearly. Account Y, however, adds a percentage of the current balance, which grows monthly, so account Y grows exponentially.

Getting to the Answer: Account X begins with \$500 (the y -intercept, or b) and adds 2% of \$500, or $\$500 \times 0.02 = \10 (the rate of change, or m), each month for 3 years, which is 36 months (the input, or x). Plug these values into the linear growth equation to solve for the final value of the account:

$$\begin{aligned}y &= mx + b \\y &= 10(36) + 500 \\y &= 360 + 500 = \$860\end{aligned}$$

Account Y begins with \$500 ($f(0)$) and adds 2%, or 0.02, (r) each month for 36 months (t). Plug these values into the exponential growth equation to solve for the final value of the account:

$$\begin{aligned}f(t) &= f(0)(1+r)^t \\f(t) &= 500(1+0.02)^{36} \\f(t) &= 500(1.02)^{36} \approx \$1,019.94\end{aligned}$$

The positive difference between the two accounts is therefore $\$1,019.94 - \$860 = \$159.94$. Round up to the nearest dollar, and enter **160**.

31. A**Difficulty:** Medium**Category:** Advanced Math

Getting to the Answer: There are two variables and only one equation, but because you're asked to solve for one of the variables *in terms of* the other, you solve the same way you would any other equation, by isolating x on one side of the equation. Cross-multiplying is a quick route to the solution:

$$\begin{aligned}\frac{6}{x} &= \frac{3}{k+2} \\6(k+2) &= 3x \\6k + 12 &= 3x \\\frac{6k}{3} + \frac{12}{3} &= \frac{3x}{3} \\2k + 4 &= x\end{aligned}$$

Switch x to the left side of the equation and the result matches **(A)**.

32. 3

Difficulty: Hard**Category:** Advanced Math

Getting to the Answer: Because the expression is adding fractions with different denominators, you'll need to establish a common denominator. Note that the second fraction is divisible by 3, so you can simplify the expression and then create the common denominator. Since both fractions now have denominators involving $(a - 3)$, wait to substitute 6 for $(a - 3)^2$ until you've added the two fractions.

$$\begin{aligned} & \frac{3a+9}{(a-3)^2} + \frac{-3}{a-3} \\ &= \frac{3a+9}{(a-3)^2} + \frac{-3}{a-3} \times \frac{a-3}{a-3} \\ &= \frac{3a+9}{(a-3)^2} + \frac{-3a+9}{(a-3)^2} \\ &= \frac{18}{(a-3)^2} \end{aligned}$$

The question specifies that $(a - 3)^2 = 6$, so $\frac{18}{(a-3)^2} = \frac{18}{6} = 3$. Therefore, the expression equals 3. Enter **3**.

33. C

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: The denominator of the expression contains the sum of two fractions that themselves have different denominators, so start by finding a common denominator:

$$\frac{\frac{2}{a}}{\frac{a-6}{(a-2)(a-6)} + \frac{a-2}{(a-2)(a-6)}} = \frac{\frac{2}{a}}{\frac{2a-8}{a^2-8a+12}}$$

Next, multiply the numerator of the expression by the reciprocal of the denominator and simplify:

$$\begin{aligned} & \frac{2}{a} \times \frac{a^2-8a+12}{2a-8} \\ &= \frac{2(a^2-8a+12)}{2a^2-8a} \\ &= \frac{a^2-8a+12}{a^2-4a} \end{aligned}$$

This expression matches **(C)**.

34. A

Difficulty: Easy**Category:** Advanced Math

Getting to the Answer: Both terms in the numerator share a common x^2 term, so factor that out:

$$\frac{x^2(x-3)}{x-3} = 9$$

Now, cancel $(x - 3)$ in the numerator and the denominator to get $x^2 = 9$. Normally, this would mean that x could be 3 or -3 . However, remember to check for extraneous solutions. If $x = 3$, then the denominator in the original equation would be equal to 0; thus, $x = -3$, and **(A)** is correct.

35. B

Difficulty: Hard**Category:** Advanced Math

Getting to the Answer: Because the question states that the expressions are equivalent, set up the equation $\frac{16}{7x+4} + A = \frac{49x^2}{7x+4}$ and solve for A . Start by subtracting the first term from both sides of the equation to isolate A . Then, simplify. The denominators of the rational terms are the same, so they can be combined. Then, cancel common factors.

$$\begin{aligned} \frac{16}{7x+4} + A &= \frac{49x^2}{7x+4} \\ A &= \frac{49x^2}{7x+4} - \frac{16}{7x+4} \\ A &= \frac{49x^2 - 16}{7x+4} \\ A &= \frac{\cancel{(7x+4)}(7x-4)}{\cancel{7x+4}} \\ A &= 7x - 4 \end{aligned}$$

The correct choice is **(B)**.

36. B

Difficulty: Medium**Category:** Advanced Math

Getting to the Answer: Move the second fraction over to the other side of the equation by subtracting it from both sides, then cross-multiply to simplify:

$$\begin{aligned} \frac{1-2c}{3c} &= \frac{c-8}{12} \\ 12(1-2c) &= 3c(c-8) \\ 12-24c &= 3c^2-24c \\ 12 &= 3c^2 \\ c^2 &= 4 \end{aligned}$$

Therefore, either $c = -2$ or $c = 2$. The question specifies that $c < 0$, so c must equal -2 . **(B)** is correct.