YII 数学 SAT 演習

●7-3 Probability Problems

1. A bag contains three green marbles, four blue marbles, and two orange marbles. If a marble is picked at random, what is the probability that an orange marble will NOT be picked?

(A) $\frac{1}{4}$
(B) $\frac{1}{3}$
(C) $\frac{4}{11}$
(D) $\frac{1}{2}$
(E) $\frac{7}{9}$

1, 2, 2, 3, 3, 3, 4, 4, 4, 4

2. What is the probability that a number selected at random from the set of numbers above will be the average of the set?

(A) 0
(B) $\frac{1}{10}$
(C) $\frac{1}{5}$
(D) $\frac{3}{10}$
(E) $\frac{2}{5}$

3. If the probability that an event will occur is $\frac{x}{4}$ and $x \neq 0$, what is the probability that this event will NOT occur ?

(A) $\frac{1-x}{4}$
(B) $\frac{4-x}{4}$
(C) $\frac{4-x}{x}$
(D) $\frac{4}{x}$
(E) $\frac{x}{x-4}$

Questions 4 and 5.

A jar contains only red, blue, and green marbles. The probability of picking a red marble is 0.25, and the probability of picking a blue marble is 0.40.

4. What is the probability of picking a green marble?

(A) 0.25
(B) 0.35
(C) 0.60
(D) 0.75
(E) 1.00

5. What is the LEAST number of marbles that could be in the jar?

(A) 4
(B) 5
(C) 10
(D) 20
(E) It cannot be determined from the information given.

6. A six-sided cube whose faces are numbered from 1 to 6 is rolled twice. Which of the following statements must be true?

I. There are 36 possible outcomes.
II. The probability that the two numbers rolled do *not* match is $\frac{1}{2}$.
III. The probability that the sum of the two numbers rolled is 5 is $\frac{1}{9}$.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

7. If the letters L, O, G, I, and C are randomly arranged to form a five-letter "word," what is the probability that the result will be the word LOGIC?

(A) $\frac{1}{120}$

(B) $\frac{1}{24}$

(C) $\frac{1}{5}$

(D) $\frac{1}{4}$

(E) $\frac{1}{2}$

8.

The circle above is divided into eight sectors of equal area. What is the probability that the spinner will land on an even-numbered region in each of two consecutive spins?

(A) $\frac{3}{4}$

(B) $\frac{25}{64}$

(C) $\frac{3}{8}$

(D) $\frac{1}{4}$

(E) $\frac{9}{64}$

9. Three fair coins are tossed at the same time. What is the probability that all three coins will come up heads *or* all will come up tails?

(A) $\frac{1}{8}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{3}{8}$

10. The faces of a red cube and a yellow cube are numbered from 1 to 6. Both cubes are rolled. What is the probability that the top face of each cube will have the same number?

(A) $\frac{1}{8}$

(B) $\frac{1}{6}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

11. A jar contains 54 marbles each of which is blue, green, or white. The probability of selecting a blue marble at random from the jar is $\frac{1}{3}$, and the probability of selecting a green marble at random is $\frac{4}{9}$. How many white marbles does the jar contain?

(A) 6
(B) 8
(C) 9
(D) 12
(E) 18

$A = \{-2, -1, 0, 1, 2\}$

12. If x represents a number picked at random from set A above, what is the probability that $x^2 < 2$?

(A) $\frac{1}{5}$

(B) $\frac{2}{5}$

(C) $\frac{3}{5}$

(D) $\frac{4}{5}$

(E) 1

13. If x represents a number picked at random from the set $\{-3, -2, -1, 0, 1, 2\}$, what is the probability that x will satisfy the inequality $4 - 3x < 6$?

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) $\frac{2}{3}$

(E) $\frac{5}{6}$

14. $A = \{1, 2, 3\}$
$B = \{1, 4, 9\}$

A number is randomly selected from set A above, and then a second number is randomly selected from set B. What is the probability that the product of the two numbers selected will be less than 9?

(A) $\frac{1}{3}$

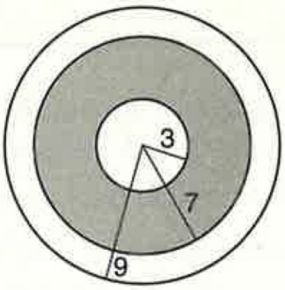
(B) $\frac{5}{9}$

(C) $\frac{2}{3}$

(D) $\frac{7}{9}$

(E) $\frac{5}{6}$

15.



A target shown in the accompanying diagram consists of three circles with the same center. The radii of the circles are 3, 7, and 9. A dart is thrown and lands on the target. What is the probability that the dart will land on the shaded region?

(A) $\frac{4}{9}$

(B) $\frac{40}{81}$

(C) $\frac{49}{81}$

(D) $\frac{7}{10}$

(E) $\frac{40}{49}$

16. The six faces of a cube are numbered 1 through 6. If the cube will be rolled twice in succession, what is the probability that the number that faces up on the first roll will be greater than the number that faces up on the second roll?

(A) $\frac{1}{4}$

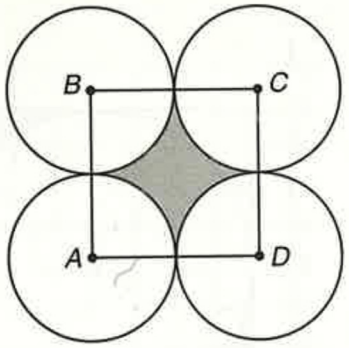
(B) $\frac{1}{3}$

(C) $\frac{5}{12}$

(D) $\frac{7}{12}$

(E) $\frac{2}{3}$

17.



In the accompanying diagram, points A, B, C, and D are the centers of four circles that each have a radius length of 1. The circles are tangent at the points shown. If a point is selected at random from the interior of square $ABCD$, what is the probability that the point will be chosen from the shaded region?

(A) $1 - \frac{\pi}{4}$

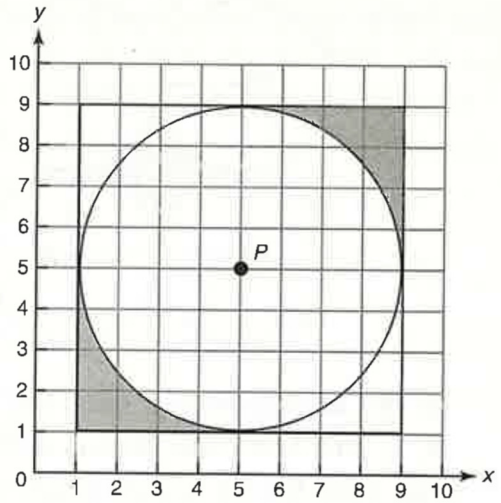
(B) $1 - \frac{3\pi}{16}$

(C) $1 - \frac{\pi}{6}$

(D) $1 - \frac{\pi}{8}$

(E) $1 - \frac{\pi}{16}$

18.



In the accompanying diagram, circle P is inscribed in a square. If a point is picked at random from the interior of the square, what is the probability that the point will lie in the shaded region?

(A) $\frac{4 - \pi}{16}$

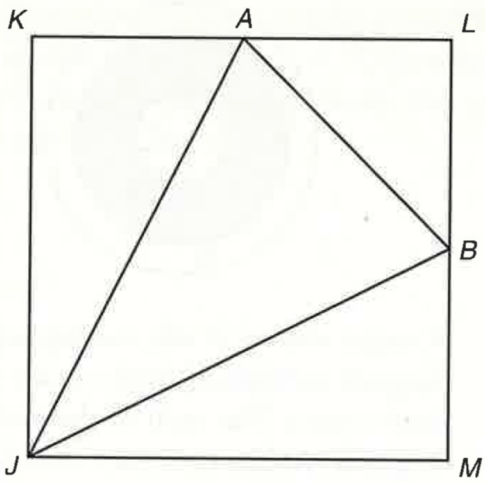
(B) $\frac{4 - \pi}{8}$

(C) $\frac{8 - \pi}{32}$

(D) $\frac{4 - \pi}{4}$

(E) $\frac{9 - 2\pi}{9}$

19.



In the accompanying figure, $JKLM$ is a square with sides of length 6. Points A and B are the midpoints of sides $\overline{KL}$ and $\overline{LM}$, respectively. If a point is selected at random from the interior of the square, what is the probability that the point will be chosen from the interior of $\triangle JAB$?

(A) $\frac{3}{16}$

(B) $\frac{1}{4}$

(C) $\frac{5}{16}$

(D) $\frac{3}{8}$

(E) $\frac{1}{2}$

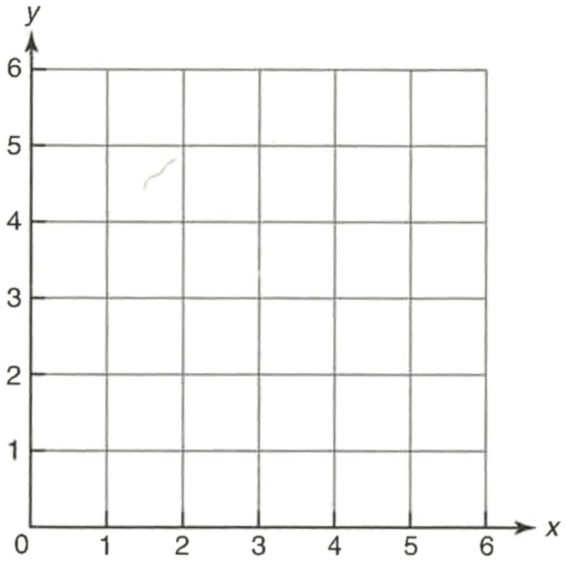
Grid-In

1. The probability of selecting a green marble at random from a jar that contains only green, white, and yellow marbles is $\frac{1}{4}$. The probability of selecting a white marble at random from the same jar is $\frac{1}{3}$. If this jar contains 10 yellow marbles, what is the total number of marbles in the jar?

2. Each of five identical cards is numbered on one of its sides with a different integer from 1 to 5. The cards are turned over and then shuffled so that they are not in a predictable order. If two cards are selected at random without replacement, what is the probability that their sum is at least 8?

3. 

In the accompanying diagram of a square dartboard, the length of a side of the larger square is 1.5 times the length of a side of the smaller square. If a dart is thrown and lands on the larger square, what is the probability that it will land in the interior of the smaller square?

4. 

A square dartboard is placed in the first quadrant from $x = 0$ to 6 and $y = 0$ to 6, as shown in the accompanying figure. A triangular region on the dartboard is enclosed by the graphs of the equations $y = 2$, $x = 6$, and $y = x$ (not shown). Find the probability that a dart that randomly hits the dartboard will land in the triangular region formed by the three lines.

5. Five students, all of different heights, are to be randomly arranged in a line. What is the probability that the tallest student will be first in line and the shortest student will be last in line?

6. A jar contains four blue marbles and two green marbles. Without looking, two marbles are drawn from the jar. What is the probability that two marbles with the same color will be selected?

●7-3 Probability Problems 解答・解説

1. (**E**) The total number of marbles in the bag is 3 + 4 + 2 or 9. Of these 9 marbles, 3 + 4 or 7 marbles are not orange. Hence, the probability that an orange marble will NOT be picked is $\frac{7}{9}$.

2. (**D**) The average of the set 1, 2, 2, 3, 3, 3, 4, 4, 4, 4 is the sum of the 10 numbers divided by 10 or $\frac{30}{10} = 3$. Since three of the 10 numbers are 3, the probability that a number selected at random from the set of 10 numbers will be the average of the set is $\frac{3}{10}$.

3. (**B**) If the probability that an event will occur is $\frac{x}{4}$ and $x \neq 0$, then the probability that this event will NOT occur is

$$1 - \frac{x}{4} = \frac{4}{4} - \frac{x}{4} = \frac{4 - x}{4}$$

4. (**B**) Since the probability of picking a red marble is 0.25 and the probability of picking a blue marble is 0.40, the probability of picking a green marble is 1 − (0.25 + 0.40) or 0.35.

5. (**D**) To find the LEAST number of marbles that could be in the jar, multiply each answer choice by 0.25, 0.40, and 0.35 until an integer is obtained for each product. Since 4 times 0.40 is 1.6, rule out choice (A); 5 times 0.25 is 1.25, so rule out choice (B); 10 times 0.35 is 3.5, so rule out choice (C). Since 20 times 0.25 is 5, 20 times 0.40 is 8, and 20 times 0.35 is 7, choice (D) is correct.

6. (**D**) Examine each Roman numeral statement in turn:
   - I. Since there are six possible outcomes for each roll of the cube, there is a total of 6 × 6 = 36 outcomes. Statement I is true.
   - II. There are six outcomes for which the two numbers rolled match: (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6). Hence, there must be 36 − 6 = 30 outcomes for which the two numbers rolled do *not* match. Hence, the probability of rolling two numbers that do not match is $\frac{30}{36} = \frac{5}{6}$. Statement II is false.
   - III. There are four outcomes for which the sum of the numbers rolled is 5: (1, 4), (4, 1), (2, 3), and (3, 2). Thus, the probability that the sum of the two numbers rolled is 5 is $\frac{4}{36} = \frac{1}{9}$. Statement III is true.

   Since only Roman numeral statements I and III are true, the correct choice is (D).

7. (**A**) The five letters *L*, *O*, *G*, *I*, and *C* can be arranged in 5 × 4 × 3 × 2 × 1 or 120 different ways. Since only one of these arrangements is "LOGIC," the probability that a random arrangement of the five letters will form the word "LOGIC" is $\frac{1}{120}$.

8. (**E**) Since three of the eight sectors are even-numbered, the probability that the spinner will land on an even-numbered region is $\frac{3}{8}$. Hence, the probability that the spinner will land on an even-numbered region in each of two consecutive spins is $\frac{3}{8} \times \frac{3}{8}$ or $\frac{9}{64}$.

9. (**C**) If three fair coins are tossed at the same time, the probability that all three will come up heads is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{8}$. The probability that all three coins will come up tails is also $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ or $\frac{1}{8}$. Hence, the probability that all three coins will come up heads *or* all will come up tails is $\frac{1}{8} + \frac{1}{8}$ or $\frac{1}{4}$.

10. (**B**) The probability that the red and the yellow cube will each show a given number from 1 to 6 is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Hence, the probability that both cubes will show a 1 or a 2 or a 3 or a 4 or a 5 or a 6 is

$$\frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{6}{36} = \frac{1}{6}$$

11. (**D**) Since the probability of selecting a blue marble at random from the jar is $\frac{1}{3}$ and the probability of selecting a green marble at random from the jar is $\frac{4}{9}$, the probability of selecting a white marble is

$$1 - \left(\frac{1}{3} + \frac{4}{9}\right) = 1 - \frac{7}{9} = \frac{2}{9}$$

If x marbles of the 54 marbles in the jar are white, then

$$\frac{x}{54} = \frac{2}{9} \qquad 9x = 108$$

$$x = \frac{108}{9} = 12$$

12. (**C**) The squares of three of the five numbers in the set $A = \{-2, -1, 0, 1, 2\}$ are less than 2:

$(-1)^2 = 1 < 2, \quad 0^2 = 0 < 2, \quad 1^2 = 1 < 2$

Hence, the probability of picking a number x from set A such that $x^2 < 2$ is $\frac{3}{5}$.

13. (**C**) If $4 - 3x < 6$, then $-3x < 2$, so $\frac{-3x}{-3} > \frac{2}{-3}$ and $x > -\frac{2}{3}$. Three of the six numbers in $\{-3, -2, -1, 0, 1, 2\}$ are greater than $-\frac{2}{3}$: 0, 1, and 2. Hence, the required probability is $\frac{3}{6}$ or $\frac{1}{2}$.

14. (**B**) If a number is randomly selected from set $A = \{1, 2, 3\}$ and then a second number is randomly selected from set $B = \{1, 4, 9\}$, the set of all possible products is as follows:

$1 \times 1 = 1 \quad 2 \times 1 = 2 \quad 3 \times 1 = 3$
$1 \times 4 = 4 \quad 2 \times 4 = 8 \quad 3 \times 4 = 12$
$1 \times 9 = 9 \quad 2 \times 9 = 18 \quad 3 \times 9 = 27$

Since five of the nine possible products are less than 9, the probability that the product of the two numbers selected will be less than 9 is $\frac{5}{9}$.

15. (**B**) To find the probability that the dart will land on the shaded region, find the ratio of the area of the shaded region to the area of the target:

$$P(\text{dart lands on target}) = \frac{\text{Area of shaded region}}{\text{Area of largest circle}} = \frac{(\pi \times 7^2) - (\pi \times 3^2)}{\pi \times 9^2} = \frac{49\pi - 9\pi}{81\pi} = \frac{40\cancel{\pi}}{81\cancel{\pi}} = \frac{40}{81}$$

16. (**C**) The total number of possible outcomes is $6 \times 6 = 36$. Count the number of favorable outcomes by making an organized list:

(6, 5), (6, 4) . . . , (6, 1)	5 outcomes
(5, 4), (5, 3), . . . , (5, 1)	4 outcomes
(4, 3), (4, 2), (4, 1)	3 outcomes
(3, 2), (3, 1)	2 outcomes
(2, 1)	1 outcome
Total favorable outcomes:	15

Calculate the probability ratio:

$$P(\text{first roll} > \text{second roll}) = \frac{15}{36} = \frac{5}{12}.$$

17. (**A**) To find the probability that the point will be chosen from the shaded region, find the ratio of the area of the shaded region to the area of square $ABCD$.

- Since the circles have the same radius, the area of the four quarter circles is equal to the area of one circle, which is $\pi \times 1^2 = \pi$.
- The length of a side of the square is $1 + 1 = 2$, so its area is $2 \times 2 = 4$.
- The area of the shaded region is the difference between the areas of the square and the four quarter circles, which is $4 - \pi$.
- Therefore:

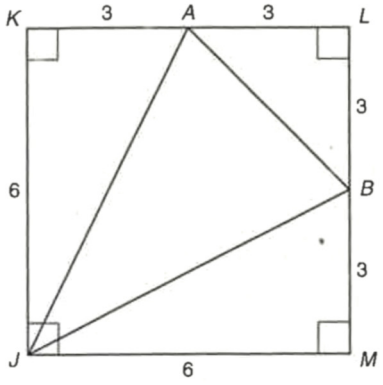
$$P(\text{Point chosen from shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of square } ABCD} = \frac{4 - \pi}{4} = \frac{4}{4} - \frac{\pi}{4} = 1 - \frac{\pi}{4}$$

18. (**B**) To find the probability that the point will be chosen from the shaded region, find the ratio of the area of the shaded region to the area of the square.

- The length of a side of the square is $9 - 1 = 8$, so its area is $8 \times 8 = 64$.
- The diameter of the inscribed circle is equal to the length of the side of the square. Hence, the radius of the inscribed circle is $\frac{1}{2} \times 8 = 4$, so the area of the inscribed circle is $\pi \times 4^2 = 16\pi$.
- The difference between the areas of the square and the inscribed circle is $64 - 16\pi$, which represents the sum of the areas of the four corners of the square bounded by the square and the circle. Since only two of these four corner regions are shaded, the area of the shaded region is $\frac{1}{2}(64 - 16\pi) = 32 - 8\pi$.
- Therefore:

$$P(\text{Point chosen from shaded region}) = \frac{\text{Area of shaded region}}{\text{Area of square}} = \frac{32 - 8\pi}{64} = \frac{\cancel{8}(4 - \pi)}{\cancel{64}_{8}} = \frac{4 - \pi}{8}$$

19. (**D**) To find the probability that the point will be chosen from the interior of $\triangle JAB$, find the ratio of the area of $\triangle JAB$ to the area of square *JKLM*. Find the area of $\triangle JAB$ indirectly by subtracting the sum of the areas of right triangles *JKA*, *BLA*, and *JMB* from the area of square *JKLM*.



$$\text{Area of } \triangle JKA = \frac{1}{2} \times 6 \times 3 = 9$$

$$\text{Area of } \triangle BLA = \frac{1}{2} \times 3 \times 3 = 4.5$$

$$\text{Area of } \triangle JMB = \frac{1}{2} \times 6 \times 3 = 9$$

$$\text{Area of } \triangle JAB = (6 \times 6) - (9 + 4.5 + 9) = 13.5$$

Hence,

$$P(\text{Point chosen from } \triangle JAB) = \frac{\text{Area of } \triangle JAB}{\text{Area of square } JKLM} = \frac{13.5}{36} = \frac{135}{360} = \frac{135 \div 45}{360 \div 45} = \frac{3}{8}$$

GRID-IN

1. **(24)** Since

$$1 - \left(\frac{1}{4} + \frac{1}{3}\right) = 1 - \frac{7}{12}$$

the probability of selecting a yellow marble is $\frac{5}{12}$. If 10 of the x marbles in the jar are yellow, then $\frac{5}{12} = \frac{10}{x}$. Since 10 is two times 5, x must be two times 12 or 24.

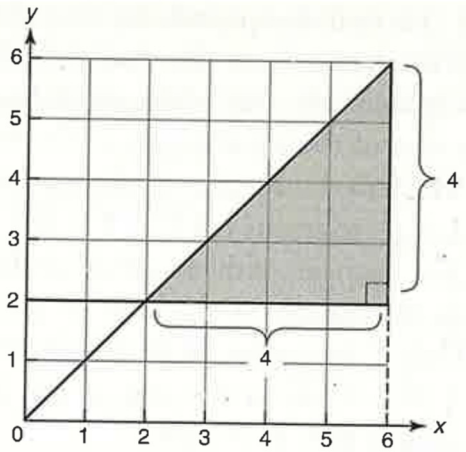
2. **(4/20)** If two cards are selected at random without replacement from a set of five identical cards each numbered on one side with a different integer from 1 to 5, the set of all possible sums is as follows:

$1+2=3$ $2+1=3$ $3+1=4$ $4+1=5$ $5+1=6$
$1+3=4$ $2+3=5$ $3+2=5$ $4+2=6$ $5+2=7$
$1+4=5$ $2+4=6$ $3+4=7$ $4+3=7$ $5+3=8$
$1+5=6$ $2+5=7$ $3+5=8$ $4+5=9$ $5+4=9$

Since four of the 20 possible sums are 8 or more, the probability of picking two cards whose sum will be at least 8 is $\frac{4}{20}$.
Grid in as 4/20.

3. **(4/9)** The probability that the dart will land in the interior of the inner square is equal to the ratio of the area of the inner square to the area of the larger square. It is given that the length of a side of the larger square is 1.5 times the length of a side of the smaller square. Suppose the length of a side of the smaller square is 2, then the length of a side of the larger square is 3 and the ratio of their areas is $\frac{2^2}{3^2} = \frac{4}{9}$.

4. **(8/36)** To find the probability that a dart will land in the triangular region formed by the lines $y = 2$, $x = 6$, and $y = x$, find the ratio of the area of the triangular region to the area of the square.
   - The area of the square dartboard is $6 \times 6 = 36$.



   - The area of the triangular region is $\frac{1}{2} \times 4 \times 4 = 8$.
   - Hence, the probability that the dart lands in the triangular region is $\frac{8}{36}$.

5. **(1/20)** To find the probability that the tallest student will be first in line and the shortest student will be last in line, find the ratio of the number of different ways in which the five students can be arranged in a line under these conditions to the number of different ways in which the five students can be arranged in a line when there are no conditions.

- The tallest student must be in the first position and the shortest student must be in the last position. Hence, each of these positions can be filled in exactly one way:

$$\boxed{1} \times \square \times \square \times \square \times \boxed{1}$$

The second position can be filled by any one of the remaining three students, the third position can be filled by any of the two remaining students, and the fourth position must be filled by the one remaining student:

$$\boxed{1} \times \boxed{3} \times \boxed{2} \times \boxed{1} \times \boxed{1} = 6$$

Under the given conditions, there are six different ways in which the students can be arranged.

- If there are no conditions, then the total number of ways in which the five students can be arranged in a line is $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.
- When the five students are randomly arranged in a line, the probability that the tallest student will be first in line and the shortest student will be last in line is $\frac{6}{120} = \frac{1}{20}$.

6. **(7/15)** The probability that two marbles with the same color will be selected is the sum of the probabilities that two blue marbles will be selected or two green marbles will be selected.
   - Of the $4 + 2 = 6$ marbles, 4 marbles are blue. Hence, the probability that two blue marbles will be selected is

$$\frac{4}{6} \times \frac{3}{5} = \frac{12}{30}$$

   - Of the 6 marbles, 2 marbles are green. Hence, the probability that two green marbles will be selected is

$$\frac{2}{6} \times \frac{1}{5} = \frac{2}{30}$$

   - Hence, the probability that two marbles with the same color will be selected is

$$\frac{12}{30} + \frac{2}{30} = \frac{14}{30} = \frac{7}{15}$$