YII 数学　SAT 演習

●7-2 Counting Problems

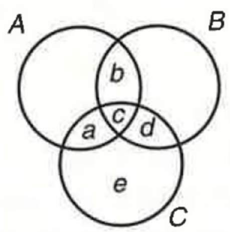
1. An ice cream parlor makes a sundae using one of six different flavors of ice cream, one of three different flavors of syrup, and one of four different toppings. What is the total number of different sundaes that this ice cream parlor can make?

(A) 72
(B) 36
(C) 30
(D) 26
(E) 13

2. Five blocks are painted red, blue, green, yellow, and white. In how many different ways can the blocks be arranged in a row if the red block is always first?

(A) 120
(B) 96
(C) 48
(D) 24
(E) 12

3.



In the figure above, five nonoverlapping regions, labeled *a* through *e*, are formed by sets *A*, *B*, and *C*. Which region represents the set of elements that belong to sets *B* and *C* but not to set *A*?

(A) *a*
(B) *b*
(C) *c*
(D) *d*
(E) *e*

4. In how many different ways can five students be seated in three chairs?

(A) 15
(B) 18
(C) 30
(D) 60
(E) 120

5. A quarter, a dime, a nickel, and a penny are placed in a box. One coin is drawn from the box and then put back before a second coin is drawn. In how many different ways can two coins be drawn so that the sum of the values of the two coins is at least 25 cents?

(A) 9
(B) 7
(C) 6
(D) 5
(E) 4

6. The students in a certain physical education class are on either the basketball team or the tennis team, are on both these teams, or are not on either team. If
15 students are on the basketball team,
18 students are on the tennis team,
11 students are on both teams, and
14 students are not on either of these teams, how many students are in the class?

(A) 48
(B) 40
(C) 36
(D) 30
(E) 28

7. How many four-digit numbers greater than 1000 can be formed from the digits 0, 1, 2, 3, 4, and 5 if the same digit cannot be used more than once?

(A) 1296
(B) 625
(C) 360
(D) 300
(E) 120

8. From the letters of the word "TRIANGLE," how many three-letter arrangements can be formed if the first letter must be T, one of the other letters must be A, and no letter can be used more than once in any arrangement?

(A) 6
(B) 10
(C) 12
(D) 42
(E) 84

9. In a French class, each student belongs to the French Club or to the Spanish Club. In this class, F students are in the French Club, S students are in the Spanish Club, and N of these students are in both the French and the Spanish clubs. What fraction of the class belongs to the French Club but not to the Spanish Club?

(A) $\frac{F-N}{F+S}$

(B) $\frac{F-N}{F+S+N}$

(C) $\frac{F-N}{F+S-N}$

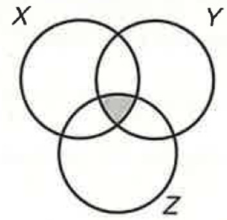
(D) $\frac{F}{F+S-2N}$

(E) $\frac{F-N}{F+S-2N}$

10. How many numbers greater than 400 but less than 9999 can be formed using the digits 0, 1, 3, 4, and 5 if no digit may be used more than once in any number?

(A) 24
(B) 48
(C) 96
(D) 120
(E) 180

11.



In the figure above, circle X represents the set of all positive odd integers, circle Y represents the set of all numbers whose square roots are integers, and circle Z represents the set of all positive multiples of 5. Which of the following numbers is a member of the set represented by the shaded region?

(A) 25
(B) 49
(C) 75
(D) 81
(E) 100

12. $X = \{1, 2, 4\}$
$Y = \{1, 3, 4\}$

If, in the sets above, x is any number in set X and y is any number in set Y, how many different values of $x + y$ are possible?

(A) Five
(B) Six
(C) Seven
(D) Eight
(E) Nine

13. In a music class of 30 students, 50% study woodwinds, 40% study strings, and 30% study both woodwinds and strings. What percent of the students in the class do NOT study either woodwind or string instruments?

(A) 25%
(B) 40%
(C) 50%
(D) 60%
(E) 75%

14. Of the 25 city council members, 12 voted for proposition *A*, 19 voted for proposition *B*, and 10 voted for both propositions. Which of the following statements must be true?

I. Seven members voted for proposition *B* but not for proposition *A*.
II. Four members voted against both propositions.
III. Eleven members voted for one proposition and against the other proposition.

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) II and III only

15. In a survey of 63 people, 33 people subscribed to magazine *A*, 30 people subscribed to magazine *B*, and 17 subscribed to magazine *C*. For any two of the magazines, 9 people subscribed to both magazines ~~but not the third.~~ If 5 people in the survey did not subscribe to any of the three magazines, how many people subscribed to all three magazines?

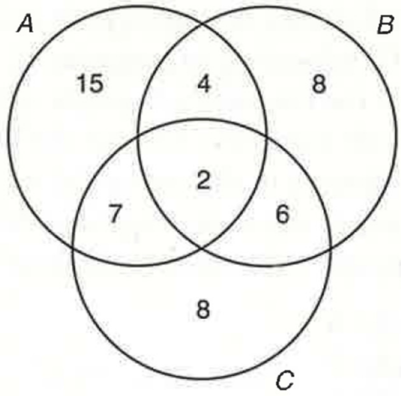
(A) 9
(B) 7
(C) 5
(D) 2
(E) It cannot be determined from the information given.

16. Six people are running in an election to fill two vacancies on a local school board. In the election booth, a voter may cast a ballot for any two of the six candidates, or cast a ballot for exactly one of the six candidates, or cast a ballot for none of the six candidates. What is the total number of different choices a voter has when casting a ballot?

(A) 9
(B) 20
(C) 22
(D) 36
(E) 37

Grid-In

1.



The accompanying diagram shows the number of students who are enrolled in three different courses. All students in circle *A* are enrolled in mathematics, all students in circle *B* are enrolled in biology, and all students in circle *C* are enrolled in computer science. What percentage of the students are enrolled in mathematics or computer science?

2. When Kim bought her new car, she found that there were 72 different ways in which her car could be equipped. Her selections included four choices of engine and three choices of transmission. If her only other selection was color, how many choices of color did she have?

3. All seven-digit telephone numbers in a certain town begin with 245. How many different telephone numbers may be assigned if the last four digits of each telephone number do not begin or end in a zero?

4. In a vacation community, 75 of the residents play golf, 67 residents play tennis, and a total of 50 residents play either golf or tennis but not both. How many residents play both golf and tennis?

●7-2 Counting Problems 解答・解説

1. **(A)** The total number of different sundaes that the ice cream parlor can make is the number of different flavors of ice cream times the number of different flavors of syrup times the number of different toppings: $6 \times 3 \times 4 = 72$.
2. **(D)** Although the red block must occupy the first position in the row, the four remaining blocks can be arranged in any order. Hence, there is 1 choice for the first position, 4 choices for the second position, 3 choices for the third position, 2 choices for the fourth position, and 1 choice for the last position. The total number of arrangements is $1 \times 4 \times 3 \times 2 \times 1$ or 24.
3. **(D)** The region in the diagram that represents the set of elements that belong to sets *B* and *C* but not to set *A* is the region in which circles *B* and *C* overlap outside of circle *A*. This region is labeled *d*.
4. **(D)** Any one of the five students can be seated in the first chair, any of the four remaining students in the second chair, and any of the three remaining students in the third chair. Thus, five students can be seated in three chairs in $5 \times 4 \times 3$ or 60 different ways.
5. **(B)** Since the first coin is replaced before another coin is picked, the same coin may be drawn twice. There are seven possible ways in which two coins whose values add up to 25 or more cents can be picked: (quarter, quarter), (quarter, dime), (dime, quarter), (quarter, nickel), (nickel, quarter), (quarter, penny), and (penny, quarter).
6. **(C)** The number of students on one but not both of the two teams is the sum of the number of students on the basketball team plus the number on the tennis team minus the number on both teams: $15 + 18 - 11 = 22$. Since 22 students are on the basketball or the tennis team and 14 students are not on either team, there are $22 + 14$ or 36 students in the class.
7. **(D)** In forming a four-digit number, 0 cannot be used as the first digit, so only five out of the six possible digits can be used to fill the first decimal position of the number. Since the same digit cannot be used more than once, any of the five remaining digits can be used to fill the second position, any of the four remaining digits can be used to fill the third position, and any of the three remaining digits can be used to fill the last position. Thus, $5 \times 5 \times 4 \times 3$ or 300 four-digit numbers greater than 1000 can be formed from the digits 0, 1, 2, 3, 4, and 5 without repeating any digit.
8. **(C)** If the first letter is *T*, as the question states, and the second letter is *A*, then any one of the six other letters can fill the last position of each three-letter arrangement. Hence, there are $1 \times 1 \times 6$ or 6 possible three-letter arrangements. If the first letter is *T* and the third letter is *A*, then any one of the six other letters can fill the middle position of each three-letter arrangement. This possibility gives $1 \times 6 \times 1$ or 6 possible three-letter arrangements. Hence, the total number of three-letter arrangements in which the first letter is *T* and one of the other letters is *A* is $6 + 6$ or 12.
9. **(C)** If *F* students in the French class are in the French Club, *S* students are in the Spanish Club, and *N* of these students are in both the French and the Spanish clubs, then $F - N$ students belong only to the French Club and the total number of students in the class is $F + S - N$. Hence, the fraction of the class that belongs to the French Club but not to the Spanish Club is $\dfrac{F-N}{F+S-N}$.

10. **(D)** When the digits 0, 1, 3, 4, and 5 are used to form a three-digit number greater than 400, the first digit must be either 4 or 5. Hence, the number of *three*-digit numbers greater than 400 that can be formed from the digits 0, 1, 3, 4, and 5, without repeating any digit, is $2 \times 4 \times 3$ or 24. When the same set of digits is used to form a four-digit number, the first digit cannot be 0. Hence, the number of *four*-digit numbers that can be formed from the digits 0, 1, 3, 4, and 5, without repeating any digit, is $4 \times 4 \times 3 \times 2$ or 96. Hence, 120 (= 24 + 96) different numbers greater than 400 but less than 9,999 can be formed.

11. **(A)** The shaded region in the diagram represents the set whose members are elements of sets *X*, *Y*, and *Z*. Look in the answer choices for a number that is odd (set *X*), has a square root that is an integer (set *Y*), and is a multiple of 5 (set *Z*). The only number that has all three properties is 25, choice (*A*).

12. **(C)** Given $X = \{1, 2, 4\}$ and $Y = \{1, 3, 4\}$, make a list of all the possible sums $x + y$:

$1 + 1 = 2$	$2 + 1 = 3$	$4 + 1 = 5$
$1 + 3 = 4$	$2 + 3 = 5$	$4 + 3 = 7$
$1 + 4 = 5$	$2 + 4 = 6$	$4 + 4 = 8$

Hence, seven different values of $x + y$ are possible: 2, 3, 4, 5, 6, 7, and 8.

13. **(B)** If in a music class of 30 students, 50% study woodwinds, 40% study strings, and 30% study both woodwinds and strings, then 50% of 30 or 15 students study woodwinds, 40% of 30 or 12 students study strings, and 30% of 30 or 9 students study both woodwinds and strings. Hence, 15 + 12 − 9 or 18 students in the class study either woodwinds or strings. Thus, 30 − 18 or 12 students do not study either woodwind or string instruments. Since

$$\frac{12}{30} = \frac{2}{5} = 0.40$$

40% of the class do not study either woodwind or string instruments.

14. **(E)** Determine whether each Roman numeral statement is true or false, given that, of 25 city council members, 12 voted for proposition *A*, 19 voted for proposition *B*, and 10 voted for both propositions.
- I. Since 19 members voted for proposition *B* and 10 voted for both propositions, 19 − 10 or 9 members voted only for proposition *B*. Hence, statement I is false.
- II. Since 12 + 19 − 10 = 21, then 21 members voted for at least one of the two propositions. Since 25 members voted, 25 − 21 or 4 members voted against both propositions. Hence, statement II is true.
- III. Since 10 members voted for both propositions, 12 − 10 or 2 members voted for proposition *A* but not for proposition *B*. Also, 19 − 10 = 9 members voted for proposition *B* but not for proposition *A*. Since 2 + 9 or 11 members voted for one proposition and against the other proposition, statement III is true.

Only Roman numeral statements II and III are true.

15. **(C)** To count the number of elements in set *A* or *B* or *C*, find the sum of the members of each set, subtract from this sum the sum of the elements common to any two of these sets, and then add the number of elements common to all three sets. If x represents the number of people who subscribe to all three magazines, the number of people who subscribe to magazine *A* or *B* or *C* is

$$33 + 30 + 17 - (9 + 9 + 9) + x \quad \text{or} \quad 53 + x$$

Since 5 of the 63 people did not subscribe to any of the three magazines, 58 people subscribed to magazine *A* or *B* or *C*. Hence, $58 = 53 + x$, so $x = 5$.

16. **(C)** Consider each of the voting choices in turn:
- A voter may cast a ballot for two of the six candidates in ${}_6C_2 = 15$ ways.
- A voter may cast a ballot for exactly one of the six candidates in 6 ways.
- A voter may cast a ballot for none of the six candidates in 1 way.

Hence, the total number of different voting choices is 15 + 6 + 1 = 22.

GRID-IN

1. **(84)** To find the percentage of students enrolled in mathematics or computer science, calculate

$$\frac{\text{number of students in math or computer science}}{\text{total number of students}} \times 100\%$$

First find the total number of students:

- The number of students taking *only* mathematics = **15**
- The number of students taking *only* computer science = **8**
- The number of students taking *only* biology = **8**
- The number of students taking mathematics *and* computer science but *not* biology = **7**
- The number of students taking mathematics *and* biology but *not* computer science = **4**
- The number of students taking computer science *and* biology but *not* mathematics = **6**
- The number of students taking mathematics *and* biology *and* computer science = **2**
- Hence, the total number of students is $15 + 8 + 8 + 7 + 4 + 6 + 2 =$ **50**

Since exactly 8 students are taking only biology, 50 − 8 = **42** students must be taking mathematics, computer science, or both mathematics and computer science. Therefore:

$$\begin{aligned}\%\text{ in math or computer science} &= \frac{42}{50} \times 100\% \\ &= 84\%\end{aligned}$$

2. **(6)** If x represents the number of color choices, then $4 \cdot 3 \cdot x = 72$, so $x = \frac{72}{12} = 6$.

3. **(8,100)** Since digits can be repeated and there are 9 digits excluding 0, there are 9 choices for the first and last digits:

$$\boxed{9} \times \boxed{} \times \boxed{} \times \boxed{9}$$

Any one of the 10 digits can be used to fill the second and third positions. Hence, the number of different telephone numbers that can be assigned when the last four digits of each telephone number does not begin or end with a zero is

$$\boxed{9} \times \boxed{10} \times \boxed{10} \times \boxed{9} = 8100$$

4. **(46)** If x represents the number of residents who play both golf and tennis, then $75 - x$ residents play only golf and $67 - x$ residents play only tennis. Hence:

$$\begin{aligned}(75 - x) + (67 - x) &= 50 \\ 142 - 2x &= 50 \\ 2x &= 142 - 50 \\ \frac{2x}{2} &= \frac{92}{2} \\ x &= 46\end{aligned}$$