

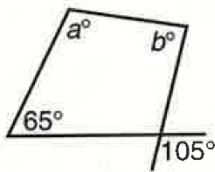
YII 数学 SAT 演習

●7-1 Average Problems

1. The average (arithmetic mean) of a set of seven numbers is 81. If one of the numbers is discarded, the average of the remaining numbers is 78. What is the value of the number that was discarded?

(A) 98
(B) 99
(C) 100
(D) 101
(E) 102

2.



In the figure above, what is the average of a and b ?

(A) 75
(B) 80
(C) 85
(D) 90
(E) 95

3. If the average of the two-digit numbers $1N$, $N7$, and NN is 35, then $N =$

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

4. What is the area of the circle whose radius is the average of the radii of two circles with areas of 16π and 100π ?

(A) 25π
(B) 36π
(C) 49π
(D) 64π
(E) 81π

5. If k is a positive integer, which of the following represents the average of 3^k and 3^{k+2} ?

(A) $\frac{1}{2} \cdot 3^{k+1}$

(B) $5 \cdot 3^k$

(C) $6^{\frac{3}{2}k}$

(D) $\frac{1}{2} \cdot 3^{3k}$

(E) $\left(\frac{3}{2}\right)^{k+1}$

6. When x is subtracted from $2y$, the difference is equal to the average of x and y . What is the value of $\frac{x}{y}$?

(A) $\frac{1}{2}$

(B) $\frac{2}{3}$

(C) 1

(D) $\frac{3}{2}$

(E) 2

7. If the average of x , y , and z is 32 and the average of y and z is 27, what is the average of x and $2x$?

(A) 42

(B) 45

(C) 48

(D) 50

(E) 63

8. For which set of numbers do the average, median, and mode all have the same value?

(A) 2, 2, 2, 2, 4

(B) 1, 3, 3, 3, 5

(C) 1, 1, 2, 5, 6

(D) 1, 1, 1, 2, 5

(E) 1, 1, 3, 5, 10

9. A man drove a car at an average rate of speed of 45 miles per hour for the first 3 hours of a 7-hour car trip. If the average rate of speed for the entire trip was 53 miles per hour, what was the average rate of speed in miles per hour for the remaining part of the trip?
- (A) 50
(B) 55
(C) 57
(D) 59
(E) 62
10. For the set of numbers 2, 2, 4, 5, and 12, which statement is true?
- (A) Mean = Median
(B) Mean > Mode
(C) Mean < Mode
(D) Mode = Median
(E) Mean = Mode
11. Susan received grades of 78, 93, 82, and 76 on four math exams. What is the lowest score she can receive on her next math exam and have an average of at least 85 on the five exams?
- (A) 96
(B) 94
(C) 92
(D) 90
(E) 88
12. What is the average of $(x + y)^2$ and $(x - y)^2$?
- (A) $\frac{x + y}{2}$
(B) xy
(C) $x^2 - y^2$
(D) $\frac{xy}{2}$
(E) $x^2 + y^2$
13. The average of the test scores of a group of x students is 76 and the average of the test scores of a group of y students is 90. When the scores of the two groups of students are combined, the average test score is 85. What is the value of $\frac{x}{y}$?
- (A) $\frac{4}{7}$
(B) $\frac{5}{9}$
(C) $\frac{2}{3}$
(D) $\frac{7}{4}$
(E) $\frac{9}{5}$
14. If the ratio of a to b is $\frac{1}{2}$ and the ratio of c to b is $\frac{1}{3}$, what is the average of a and c in terms of b ?
- (A) $\frac{b}{6}$
(B) $\frac{b}{4}$
(C) $\frac{5b}{12}$
(D) $\frac{b}{2}$
(E) $\frac{5b}{6}$
15. The average of a , b , c , d , and e is 28. If the average of a , c , and e is 24, what is the average of b and d ?
- (A) 31
(B) 32
(C) 33
(D) 34
(E) 36

16. If $2a + b = 7$ and $b + 2c = 23$, what is the average of a , b , and c ?

- (A) 5
- (B) 7.5
- (C) 10
- (D) 12.25
- (E) 15

17. The average final exam grade of 10 students is x , and the average final exam grade of 15 students is y . If the average final exam grade of the 25 students is t , what is x in terms of y and t ?

- (A) $\frac{5t - 3y}{2}$
- (B) $\frac{3t - 5y}{2}$
- (C) $\frac{3t - 2y}{5}$
- (D) $\frac{t - 15y}{10}$
- (E) $\frac{t - 10y}{15}$

18. The average of a , b , c , and d is p . If the average of a and c is q , what is the average of b and d in terms of p and q ?

- (A) $2p + q$
- (B) $2p - q$
- (C) $2q + p$
- (D) $2q - p$
- (E) $\frac{2p + q}{3}$

Grid-In

1. The average of r and s is 7.5 and the average of r , s and t is 11. What is the value of t ?
2. If the average of x , y , and z is 12, what is the average of $3x$, $3y$, and $3z$?
3. If the average of three consecutive multiples of 4 is 32, what is the LEAST of these multiples?
4. The median of seven test scores is 82, the mode is 87, the lowest score is 70, and the average is 80. If the scores are integers, what is the greatest possible test score?

●7-1 Average Problems 解答・解説



1. (B) If the average (arithmetic mean) of a set of seven numbers is 81, then the sum of these seven numbers is 7×81 or 567. Since, if one of the numbers is discarded, the average of the six remaining numbers is 78, the sum of these six numbers is 6×78 or 468. Since $567 - 468 = 99$, the value of the number that was discarded is 99.

2. (E) Since vertical angles are equal, the angle of the quadrilateral opposite the 105° angle also measures 105° . Since the sum of the measures of the angles of a quadrilateral is 360° ,

$$\begin{aligned} a + b + 105^\circ + 65^\circ &= 360^\circ \\ a + b &= 360^\circ - 170^\circ \\ \frac{a + b}{2} &= \frac{190}{2} = 95 \end{aligned}$$



3. (C) If the average of the two-digit numbers $1N$, $N7$, and NN is 35, then the sum of the three numbers is 3×35 or 105. Since $1N + N7 + NN = 105$, look for a value of N in the set of answer choices such that $N + 7 + N$ is 15 or 25. For choice (C), $N = 4$,

$$14 + 47 + 44 = 105$$

4. (C) The radii of circles with areas of 16π and 100π are 4 and 10, respectively. The average of 4 and 10 is

$$\frac{4 + 10}{2} = \frac{14}{2} = 7$$

The area of a circle of radius 7 is $\pi(7)^2$ or 49π .

5. (B) The average of 3^k and 3^{k+2} is their sum divided by 2:

$$\begin{aligned} \frac{3^k + 3^{k+2}}{2} &= \frac{3^k + 3^2 \cdot 3^k}{2} \\ &= \frac{1 \cdot 3^k + 9 \cdot 3^k}{2} \\ &= \frac{10 \cdot 3^k}{2} \\ &= 5 \cdot 3^k \end{aligned}$$

6. (C) When x is subtracted from $2y$ the difference is equal to the average of x and y , so $2y - x = \frac{x + y}{2}$ or $2(2y - x) = x + y$. Hence,

$$4y - 2x = x + y$$

$$\begin{aligned} 4y - y &= x + 2x \\ 3y &= 3x \end{aligned}$$

Dividing both sides of $3y = 3x$ by $3y$ gives $\frac{x}{y} = 1$.

7. (E) If the average of x , y , and z is 32, then

$$\frac{x + y + z}{3} = 32, \text{ so}$$

$$x + y + z = 3 \times 32 = 96$$

Since the average of y and z is 27, then

$$\frac{y + z}{2} = 27, \text{ so}$$

$$y + z = 2 \times 27 = 54$$

Substituting the value of $y + z$ in the equation $x + y + z = 96$ gives $x + 54 = 96$, so

$$x = 96 - 54 = 42$$

and

$$2x = 84$$

Hence, the average of x and $2x$ equals

$$\frac{42 + 84}{2} = \frac{126}{2} = 63$$

8. (B) Eliminate choice (A) because the mode is 2 but the average (mean) of 2, 2, 2, 2, and 4 must be greater than 2. Eliminate choices (C), (D), and (E) because the mode in each case is 1 but the average (mean) in each of these answer choices must be greater than 1. In choice (B) the mode is 3, the average (mean) of 1, 3, 3, 3, and 5 is

$$\frac{1 + 3 + 3 + 3 + 5}{5} = \frac{15}{5} = 3$$

and the median is 3 since two values (1 and 3) are less than or equal to 3 and two other values (3 and 5) are equal to or greater than 3.

9. (D) Let x represent the average rate of speed in miles per hour for the second part of the trip, which lasts $7 - 3$ or 4 hours. Since rate multiplied by time equals distance, the man drives 45×3 or 135 miles during the first part of the trip and $4x$ miles during the second part. Total distance divided by total time gives the average rate of speed for the entire trip, so

$$\frac{135 + 4x}{7} = 53$$

$$135 + 4x = 7 \times 53 = 371$$

$$4x = 371 - 135 = 236$$

$$x = \frac{236}{4} = 59$$

10. (B) For the set of numbers 2, 2, 4, 5, and 12, the mode is 2, the mean (or average) is

$$\frac{2 + 2 + 4 + 5 + 12}{5} = \frac{25}{5} = 5$$

and the median or middle value in the list is 4. Hence, the mean is greater than the mode.

11. (A) If x represents the lowest score Susan can receive on her next math exam and have an average of at least 85 on the five exams, then

$$\begin{aligned} \frac{78 + 93 + 82 + 76 + x}{5} &= 85 \\ 329 + x &= 5 \times 85 \\ &= 425 \\ x &= 425 - 329 \\ &= 96 \end{aligned}$$

12. (E) Since

$$(x + y)^2 = x^2 + 2xy + y^2$$

and

$$(x - y)^2 = x^2 - 2xy + y^2$$

the average of $(x + y)^2$ and $(x - y)^2$ is

$$\frac{(x^2 + 2xy + y^2) + (x^2 - 2xy + y^2)}{2}$$

or

$$\frac{2(x^2 + y^2)}{2} = x^2 + y^2$$

13. (B) The sum of the scores of the group of x students is $76x$ and the sum of the scores of the group of y students is $85y$. Since there are $x + y$ students in the combined group, the average of the combined group is

$$\frac{76x + 90y}{x + y}. \text{ Hence: } \frac{76x + 90y}{x + y} = 85$$

$$\begin{aligned} 76x + 90y &= 85(x + y) & \frac{5y}{y} &= \frac{9x}{y} \\ 76x + 90y &= 85x + 85y & \frac{5}{9} &= \frac{9x}{9y} \\ 90y - 85y &= 85x - 76x & \frac{5}{9} &= \frac{x}{y} \end{aligned}$$

14. (C) If the ratio of a to b is $\frac{1}{2}$, then $\frac{a}{b} = \frac{1}{2}$, so $2a = b$ or $a = \frac{b}{2}$. Since the ratio of c to b is $\frac{1}{3}$, then $\frac{c}{b} = \frac{1}{3}$, so $3c = b$ or $c = \frac{b}{3}$. Hence, the average of a and c in terms of b is

$$\frac{1}{2}(a + c) = \frac{1}{2}\left(\frac{b}{2} + \frac{b}{3}\right) = \frac{1}{2}\left(\frac{5b}{6}\right) = \frac{5b}{12}$$

15. (D) If the average of a , b , c , d , and e is 28, then

$$\frac{a + b + c + d + e}{5} = 28$$

or

$$a + b + c + d + e = 5 \times 28 = 140$$

If the average of a , c , and e is 24, then

$$\frac{a + c + e}{3} = 24$$

or

$$a + c + e = 3 \times 24 = 72$$

Substituting 72 for $a + c + e$ in $a + b + c + d + e = 140$ gives

$$b + d + 72 = 140$$

$$b + d = 140 - 72 = 68$$

$$\frac{b + d}{2} = \frac{68}{2} = 34$$

16. (A) Adding corresponding sides of the given equations, $2a + b = 7$ and $b + 2c = 23$, gives $2a + 2b + 2c = 30$ or $a + b + c = 15$. Hence:

$$\frac{a + b + c}{3} = \frac{15}{3} = 5$$

17. (A) If the average final exam grade of 10 students is x , then the sum of the final exam grades of the 10 students is $10x$. Similarly, the sum of the final exam grades of 15 students is $15y$. If the average final exam grade of the 25 students is t , then

$$\frac{10x + 15y}{25} = t$$

$$10x + 15y = 25t$$

$$2x + 3y = 5t$$

$$2x = 5t - 3y$$

$$x = \frac{5t - 3y}{2}$$

18. (B) The average of a , b , c , and d is p , so



$$\frac{a + b + c + d}{4} = p$$

or $a + b + c + d = 4p$. Similarly, since the average of a and c is q , then $a + c = 2q$, so $b + d + 2q = 4p$. Since $b + d = 4p - 2q$, then

$$\frac{b + d}{2} = \frac{4p}{2} - \frac{2q}{2} = 2p - q$$

GRID-IN

1. (18) It is given that $\frac{r+s}{2} = 7.5$, so $r + s = 2(7.5) = 15$. It is also given that $\frac{r+s+t}{3} = 11$, so $r + s + t = 3(11) = 33$. Hence, $15 + t = 33$, so $t = 33 - 15 = 18$.

2. (36) *Solution 1:* Since the average of x , y , and z is 12, then $x + y + z = 3 \times 12 = 36$. Hence,

$$3x + 3y + 3z = 3(36) = 108$$

The average of $3x$, $3y$, and $3z$ is their sum, 108, divided by 3 since three values are being added: $\frac{108}{3}$ or 36.

Solution 2: The average of x , y , and z is 12, so the average of any constant multiple of x , y , and z is 12 times that constant multiple, or $12(3) = 36$.

3. (28) If the average of three consecutive multiples of 4 is 32, then



$$\frac{4k + 4(k + 1) + 4(k + 2)}{3} = 32$$

where k is some positive integer. Hence,

$$12k + 12 = 3 \times 32$$

$$12k = 84$$

$$k = \frac{84}{12} = 7$$

The least of the three consecutive multiples is $4k = 4(7) = 28$.

4. (91) Let x represent the greatest possible test score. From the given information, at least four of the seven scores are known and two of the unknown scores must be below 82, which is given as the median:

$$70, \underline{?}, \underline{?}, 82, 87, 87, x$$

Since the average of the seven integer test scores is fixed at 80, the last test score will be greatest when the two unknown test scores below the average have the smallest possible value. The two unknown test scores below the median must be unequal since the mode is given as 87. Hence, these test scores must be 71 and 72. Since the average is 80,

$$\frac{70 + 71 + 72 + 82 + 87 + 87 + x}{7} = 80$$

$$\begin{aligned} 70 + 71 + 72 + 82 + 87 + 87 + x &= 80 \times 7 \\ 469 + x &= 560 \\ x &= 560 - 469 \\ &= 91 \end{aligned}$$