YII 数学　SAT 演習

●6-8 Coordinate Geometry

1. The length of the line segment whose endpoints are $(3,-1)$ and $(6,5)$ is

   (A) 3
   (B) 5
   (C) $3\sqrt{5}$
   (D) $5\sqrt{3}$
   (E) $\sqrt{97}$

2. What is the area of a rectangle whose vertices are $(-2,5)$, $(8,5)$, $(8,-2)$, and $(-2,-2)$?

   (A) 45
   (B) 50
   (C) 55
   (D) 60
   (E) 70

3. What is the area of a parallelogram whose vertices are $(-4,-2)$, $(-2,6)$, $(10,6)$, and $(8,-2)$?

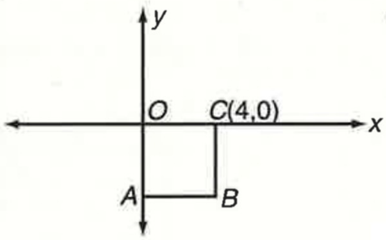
   (A) 32
   (B) 48
   (C) 72
   (D) 96
   (E) 104

4. What is the area of a triangle whose vertices are $(-4,0)$, $(2,4)$, and $(4,0)$?

   (A) 8
   (B) 12
   (C) 16
   (D) 32
   (E) 64

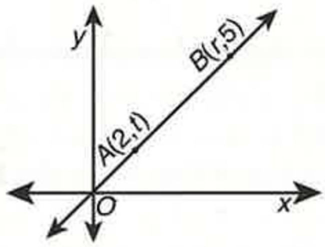
5. If $A(-3,0)$ and $C(5,2)$ are the endpoints of diagonal AC of rectangle $ABCD$, with B on the x-axis, what is the perimeter of rectangle $ABCD$?

   (A) 24
   (B) 20
   (C) 16
   (D) 14
   (E) 10

6. 

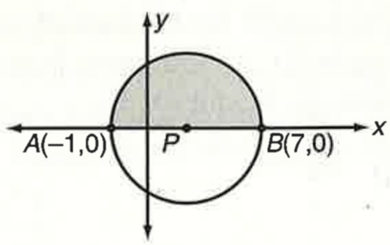
   In the figure above, if A, B, C, and O are the vertices of a square and the coordinates of A are (k, p), what are the values of k and p?

   (A) $k = -4$ and $p = 0$
   (B) $k = 0$ and $p = -4$
   (C) $k = -2$ and $p = 0$
   (D) $k = 0$ and $p = -2$
   (E) $k = 2$ and $p = -2$

7. 

   In the graph above, what is r in terms of t?

   (A) $\frac{5}{2}t$
   (B) $\frac{2}{5}t$
   (C) $\frac{t}{10}$
   (D) $10t$
   (E) $\frac{10}{t}$

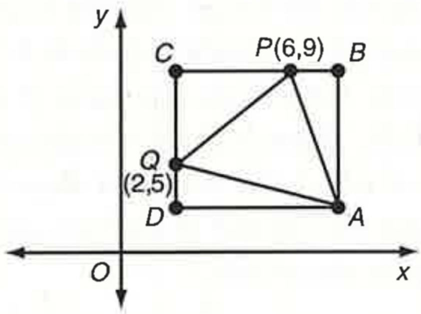
8. 

   In the figure above, if AB is a diameter of circle P, what is the perimeter of the shaded region?

   (A) $4\pi + 8$
   (B) $8\pi + 4$
   (C) $8\pi + 8$
   (D) $16\pi + 4$
   (E) $16\pi + 8$

9. The point whose coordinates are $(4,-2)$ lies on a line whose slope is $\frac{3}{2}$. Which of the following are the coordinates of another point on this line?

(A) (1,0)
(B) (2,1)
(C) (6,1)
(D) (7,0)
(E) (1,4)



Questions 10 and 11 refer to square *ABCD* shown in the figure above.

10. If the area of square *ABCD* is 36, what are the coordinates of point *A*?

(A) (7, 3)
(B) (7, 4)
(C) (8, 3)
(D) (8, 4)
(E) (9, 5)

11. What is the area of $\triangle APQ$?

(A) 24
(B) 20
(C) 18
(D) 16
(E) 12

12. If point $E(5,h)$ is on the line that contains $A(0,1)$ and $B(-2,-1)$, what is the value of h?

(A) -1
(B) 0
(C) 1
(D) 3
(E) 6

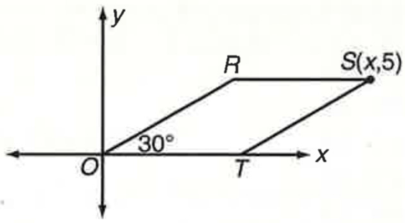
13. If the line whose equation is $y = x + 2k$ passes through point $(1,-3)$, then $k =$

(A) -2
(B) -1
(C) 1
(D) 2
(E) 4

14. A circle that has its center at the origin and passes through $(-8,-6)$ will also pass through which point?

(A) (1,10)
(B) (4,9)
(C) (7,7)
(D) $(9,\sqrt{19})$
(E) $(\sqrt{37},8)$

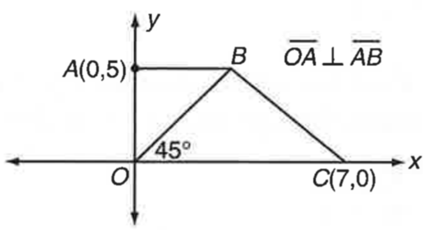
15.



In the figure above, *ORST* is a parallelogram with $OR = OT$. What is the perimeter of parallelogram *ORST*?

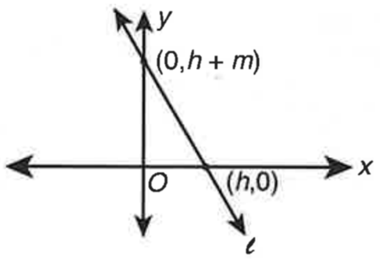
(A) 20
(B) 32
(C) $20\sqrt{3}$
(D) 40
(E) 64

16.



In the figure above, what is the area of quadrilateral *OABC*?

(A) 15
(B) 20
(C) 25
(D) 30
(E) 40

17. 

In the figure above, if the slope of line ℓ is m, what is m in terms of h?

(A) $\frac{h}{1+h}$

(B) $\frac{-h}{1+h}$

(C) $\frac{h}{1-h}$

(D) $1 + h$

(E) $1 - h$

18. Which could be the slope of a line that contains (1, 1) and passes between the points (0, 2) and (0, 3)?

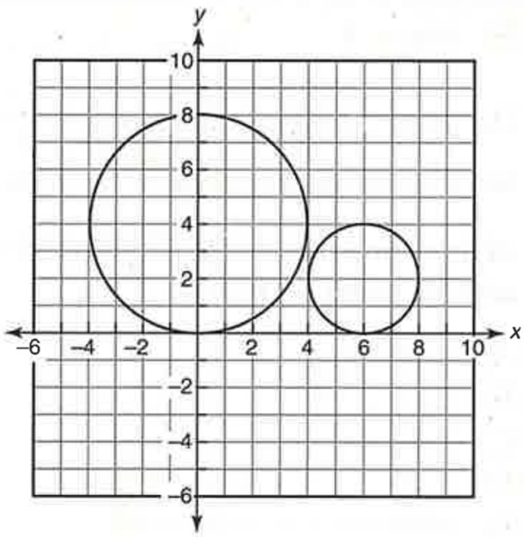
(A) $-\frac{3}{2}$

(B) $-\frac{1}{2}$

(C) 0

(D) $\frac{1}{2}$

(E) $\frac{3}{2}$

19. 

In the accompanying diagram, the diameter of the larger circle is 8 and the diameter of the smaller circle is 4. The circles are tangent to the x-axis at (0,0) and (6,0). What is the x-coordinate of the point at which the line (not shown) that contains the centers of the two circles intersects the x-axis?

(A) 10
(B) 11
(C) 12
(D) 13
(E) 14

20. The line $y + 2x = b$ is perpendicular to a line that passes through the origin. If the two lines intersect at the point $(k + 2, 2k)$, what is the value of k?

(A) $-\frac{3}{2}$

(B) $-\frac{2}{3}$

(C) $\frac{2}{5}$

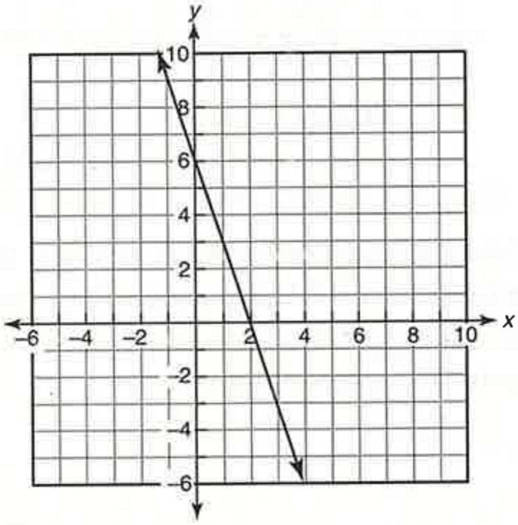
(D) $\frac{2}{3}$

(E) $\frac{3}{2}$

21. Which of the following is an equation of the line that is parallel to the line $y - 4x = 0$ and has the same y-intercept as the line $y + 3 = x + 1$?

(A) $y = 4x - 2$
(B) $y = 4x + 1$
(C) $y = -\frac{1}{4}x + 1$
(D) $y = -\frac{1}{4}x - 2$
(E) $y = -4x + 2$

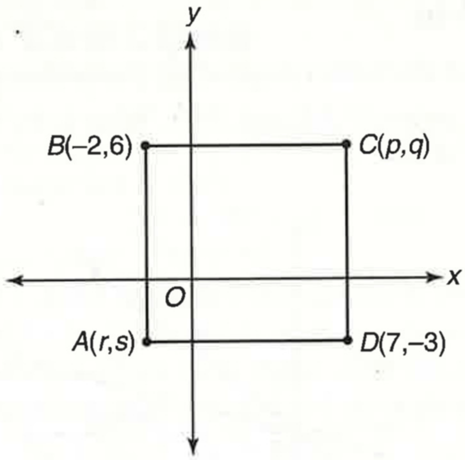
22.



Which of the following is an equation of the line shown in the accompanying figure?

(A) $y = 3x + 6$
(B) $y = -3x - 6$
(C) $y = -3x + 6$
(D) $y = -6x + 3$
(E) $y = -3x + 2$

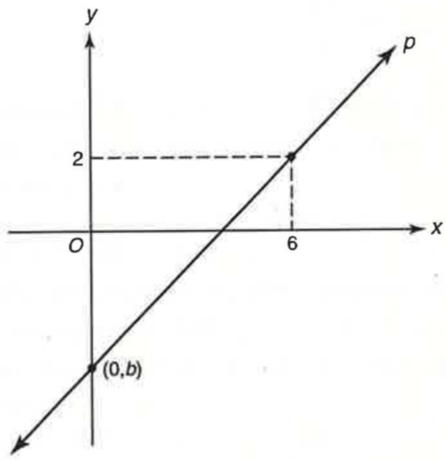
23.



Which of the following is an equation of the line that contains diagonal $\overline{AC}$ of square $ABCD$ shown in the accompanying figure?

(A) $y = 2x + 1$
(B) $y = -x + 1$
(C) $y = \frac{1}{2}x - 2$
(D) $y = 2x - 8$
(E) $y = x - 1$

24.



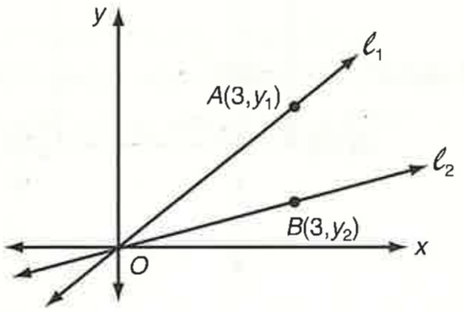
Note: Figure is not drawn to scale.

If the slope of line p shown in the figure above is $\frac{3}{2}$, what is the value of b?

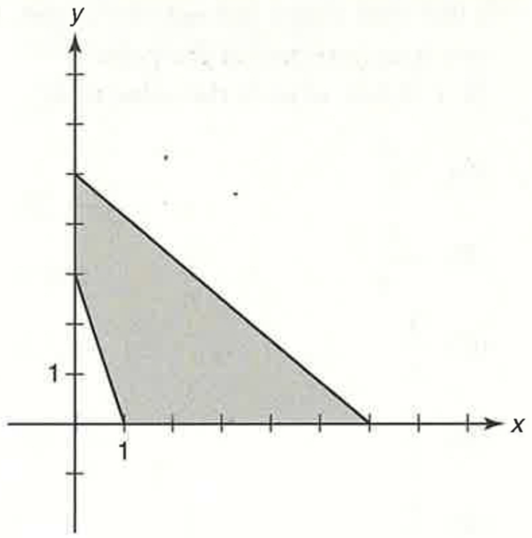
(A) −8
(B) −7
(C) −5
(D) −3
(E) −2

Grid-In

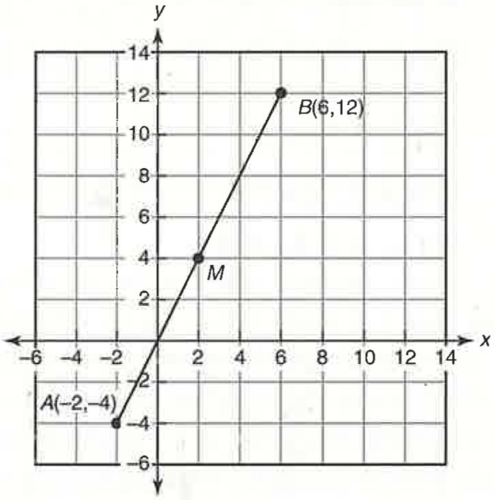
1. A line with a slope of $\frac{3}{14}$ passes through points $(7,3k)$ and $(0,k)$. What is the value of k?

2. 

In the figure above, the slope of line ℓ_1 is $\frac{5}{6}$ and the slope of line ℓ_2 is $\frac{1}{3}$. What is the distance from point A to point B?



3. In the figure above, what is the number of square units in the area of the shaded region?

4. 

In the accompanying figure, what is the y-coordinate of the point at which the line that is perpendicular to $\overline{AB}$ (not shown) at point M crosses the y-axis?

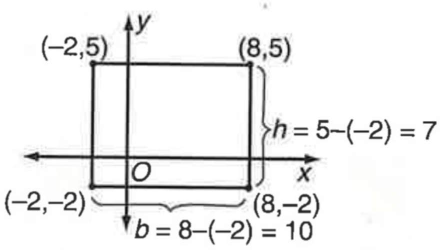
●6-8 Coordinate Geometry 解答・解説

1. **(C)** Let $(x_A, y_A) = (3,-1)$ and $(x_B, y_B) = (6,5)$. Use the distance formula to find the distance d between these points:

$$\begin{aligned} d &= \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} \\ &= \sqrt{(6-3)^2 + (5-(-1))^2} \\ &= \sqrt{3^2 + (5+1)^2} \\ &= \sqrt{9+36} \\ &= \sqrt{45} = \sqrt{9}\sqrt{5} = 3\sqrt{5} \end{aligned}$$

The length of the line segment whose endpoints are $(3,-1)$ and $(6,5)$ is $3\sqrt{5}$.

2. **(E)** Sketch the rectangle whose vertices are $(-2,5)$, $(8,5)$, $(8,-2)$, and $(-2,-2)$.



Since

$$b = 8 - (-2) = 8 + 2 = 10$$

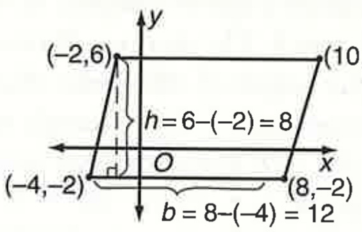
and

$$h = 5 - (-2) = 5 + 2 = 7$$

then

$$\begin{aligned} \text{Area of rectangle} &= b \times h \\ &= 10 \times 7 \\ &= 70 \end{aligned}$$

3. **(D)** Sketch the parallelogram whose vertices are $(-4,-2)$, $(-2,6)$, $(10,6)$, and $(8,-2)$.



Since

$$b = 8 - (-4) = 8 + 4 = 12$$

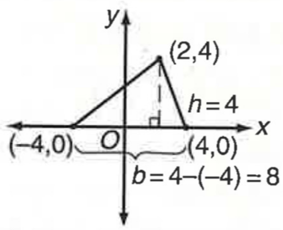
and

$$h = 6 - (-2) = 6 + 2 = 8$$

then

$$\begin{aligned} \text{Area of parallelogram} &= b \times h \\ &= 12 \times 8 \\ &= 96 \end{aligned}$$

4. **(C)** Sketch the triangle whose vertices are $(-4,0)$, $(2,4)$, and $(4,0)$.



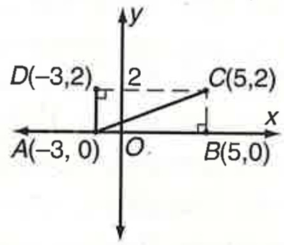
Since

$$b = 4 - (-4) = 4 + 4 = 8$$

and $h = 4$, then

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} \times b \times h \\ &= \frac{1}{2} \times 8 \times 4 \\ &= 16 \end{aligned}$$

5. **(B)** If $A(-3,0)$ and $C(5,2)$ are the endpoints of diagonal AC of rectangle $ABCD$, then the other two vertices are $(-3,2)$ and $(5,0)$.



Since the length of one side of the rectangle is 8 and the length of an adjacent side is 2, the perimeter of rectangle $ABCD$ is $2(8 + 2)$ or 20.

6. **(B)** Since point A is on the y-axis and below the x-axis, its x-coordinate is 0 and its y-coordinate is negative. You are told that $OABC$ is a square, so $OA = OC = 4$. Hence, the y-coordinate of A is -4, making $(0,-4) = (k,p)$, so $k = 0$ and $p = -4$.

7. **(E)** Since points O, A, and B lie on the same line,

$$\begin{aligned} \text{Slope } OA &= \text{Slope } OB \\ \frac{\text{Change in } y}{\text{Change in } x} = \frac{t-0}{2-0} &= \frac{5-0}{r-0} \\ \frac{t}{2} &= \frac{5}{r} \\ r \times t &= 2 \times 5 \\ r &= \frac{10}{t} \end{aligned}$$

8. (**A**) The perimeter of the shaded region is the sum of the circumference of semicircle P and the length of diameter AB. Since

$$AB = 7 - (-1) = 8$$

the diameter of circle P is 8, so

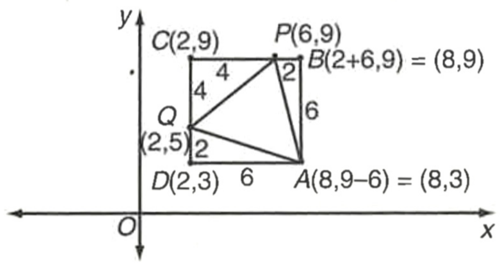
$$\begin{aligned}\text{Circumference semicircle } P &= \frac{1}{2}(\pi D)\\ &= \frac{1}{2}(8\pi)\\ &= 4\pi\end{aligned}$$

Hence, the perimeter of the shaded region is $4\pi + 8$.

9. (**C**) Test each point in the set of answer choices until you find the point that makes the slope m of the line containing that point and (4,−2) equal to $\frac{3}{2}$. Let $(x_A, y_A) = (4,-2)$. Choice (C) works since, if $(x_B, y_B) = (6,1)$, then

$$m = \frac{y_B - y_A}{x_B - x_A} = \frac{1-(-2)}{6-4} = \frac{1+2}{2} = \frac{3}{2}$$

The coordinates of another point on the line are (6,1).



10. (**C**) The coordinates of the vertices of square $ABCD$ are shown in the figure above.

11. (**D**) The area of $\triangle APQ$ is equal to the area of square $ABCD$ minus the sum of the area of the three corner triangles. Find the area of each of the corner right triangles:

- Area right $\triangle QDA$ = Area right $\triangle PBA$ $= \frac{1}{2} \times 2 \times 6 = 6$
- Area right $\triangle QCP = \frac{1}{2} \times 4 \times 4 = 8$
- Area of $\triangle APQ = 36 - (6 + 6 + 8)$ $= 36 - 20$ $= 16$

12. (**E**) Since it is given that point $E(5,h)$ is on the line that contains $A(0,1)$ and $B(-2,-1)$, Slope of $\overline{EA}$ = Slope of $\overline{AB}$

$$= \frac{-1-1}{-2-0} = \frac{-2}{-2} = 1$$

Hence,

$$\begin{aligned}\text{Slope of } \overline{EA} &= \frac{h-1}{5-0} = 1\\ h - 1 &= 5\\ h &= 6\end{aligned}$$

13. (**A**) If the line whose equation is $y = x + 2k$ passes through point (1,−3), then substituting −3 for y and 1 for x will make the resulting equation a true statement. Hence:

$$\begin{aligned}-3 &= 1 + 2k\\ -4 &= 2k\\ k &= \frac{-4}{2} = -2\end{aligned}$$

14. (**D**) If a circle with center at the origin passes through (−8,−6), the length of a radius of the circle is the length of the segment whose endpoints are $A(0,0)$ and $B(-8,-6)$. Use the distance formula to find the length of AB:

$$\begin{aligned}AB &= \sqrt{(-8-0)^2 + (-6-0)^2}\\ &= \sqrt{64 + 36}\\ &= \sqrt{100}\\ &= 10\end{aligned}$$

Any point that lies on the circle will be the same distance from the origin as point B. Find the distance from the origin to each point in the set of answer choices until you obtain a distance of 10. Choice (D) works since

$$\sqrt{(9-0)^2 + (\sqrt{19}-0)^2} = \sqrt{81 + 19} = \sqrt{100} = 10$$

The circle passes through $(9, \sqrt{19})$ since the distance of this point from the origin is 10.

15. **(D)** Drop a perpendicular to the x-axis from point S. Call the point where it intersects the x-axis point H. Since point S is 5 units above the x-axis, $SH = 5$. Opposite sides of a parallelogram are parallel, so $\angle STH = \angle O = 30°$. In 30°-60° right triangle SHT, hypotenuse ST is 2 times the length of SH (the side opposite the 30° angle). Hence, $ST = 2 \times 5 = 10$. Since opposite sides of a parallelogram have the same length, $OR = ST = 10$. Also, since $OR = OT$, then $OT = RS = 10$. Hence, the perimeter of parallelogram $ORST$ is $10 + 10 + 10 + 10$ or 40.

16. **(D)** The area of quadrilateral $OABC$ is the sum of the areas of triangles OAB and OBC.
   - Since $\angle BOC = 45$, then
   $$AOB = 90 - 45 = 45$$
   Also, since point A is 5 units above the x-axis, $OA = OB = 5$, so
   $$\text{Area right } \triangle OAB = \frac{1}{2} \times 5 \times 5 = 12.5$$
   - Since $AB \perp y$-axis, point B is 5 units above the x-axis, so the height of $\triangle OBC$ is 5 and the length of base OC is 7. Hence:
   $$\text{Area } \triangle OBC = \frac{1}{2} \times 7 \times 5 = 17.5$$
   - The area of quadrilateral $OABC$ is 12.5 + 17.5 or 30.

17. **(B)** If the slope of line ℓ is m, then
$$m = \frac{(h + m) - 0}{0 - h} = \frac{h + m}{-h}$$
Hence, $-hm = h + m$ or $-h = m + mh$. Factoring out m from the right side of the equation gives $-h = m(1 + h)$ so
$$m = \frac{-h}{1 + h}$$

18. **(A)** The slope of a line through (1, 1) and (0, 2) is $\frac{2-1}{0-1} = -1$ and the slope of a line through (1, 1) and (0, 3) is $\frac{3-1}{0-1} = -2$. Hence, the slope of a line through (1, 1) and a point between (0, 2) and (0, 3) must have a value between -1 and -2 such as $-\frac{3}{2}$.

19. **(C)** First find an equation $y = mx + b$ of the line that contains the centers of the two circles:
   - The center of the larger circle is at (0,4) and the center of the smaller circle is at (6,2). Hence, the slope of the line that contains these points is:
   $$m = \frac{\Delta y}{\Delta x} = \frac{2 - 4}{6 - 0} = -\frac{2}{6} = -\frac{1}{3}$$
   - Since the y-intercept of the line is (0,4), $b = 4$, so an equation of the line is $y = -\frac{1}{3}x + 4$.
   - The line will intersect the x-axis when $y = 0$. To find the corresponding value of x, replace y with 0 in the equation of the line. Thus, $0 = -\frac{1}{3}x + 4$, so $\frac{1}{3}x = 4$ and $x = 3 \cdot 4 = 12$.

20. **(D)** If $y + 2x = b$, then $y = -2x + b$, so the slope of this line is -2. The slope of a line perpendicular to this line is $\frac{1}{2}$, the negative reciprocal of -2. If this line passes through the origin, its y-intercept is 0, so its equation is $y = \frac{1}{2}x$. Because the point of intersection of the two lines is $(k + 2, 2k)$, the coordinates of this point must satisfy both equations. Find the value of k by substituting the coordinates of this point into the equation $y = \frac{1}{2}x$:
$$\begin{aligned} 2k &= \frac{1}{2}(k + 2) \\ 4k &= k + 2 \\ 3k &= 2 \\ \frac{3k}{3} &= \frac{2}{3} \\ k &= \frac{2}{3} \end{aligned}$$

21. **(A)** Let $y = mx + b$ represent the equation of the desired line.
   - If $y - 4x = 0$, then $y = 4x$, so the slope of this line is 4. Since parallel lines have equal slopes, $m = 4$.
   - If $y + 3 = -x + 1$, then $y = -x - 2$, so its y-intercept is -2. Since the desired line has the same y-intercept, $b = -2$.
   - The equation of the desired line is $y = 4x - 2$.

22. (**C**) From the graph, the y-intercept is at (0,6). Hence, the equation of the line has the form $y = mx + 6$. To find the slope of the line, apply the slope formula to the x- and y-intercepts: $\frac{\Delta y}{\Delta x} = \frac{6-0}{0-2} = -3$.
Since $m = -3$ and $b = 6$, an equation of the line is $y = -3x + 6$.

23. (**E**) Since $ABCD$ is a square, C has the same x-coordinate as D and the same y-coordinate as B. Hence, $p = 7$ and $q = 6$. Point A has the same x-coordinate as B and the same y-coordinate as point D. Hence, $r = -2$ and $s = -3$.
   - To find an equation of the line that contains diagonal $\overline{AC}$, first find the slope of $\overline{AC}$:

$$m = \frac{\Delta y}{\Delta x} = \frac{6-(-3)}{7-(-2)} = \frac{9}{9} = 1$$

Hence, the equation of $\overline{AC}$ has the form $y = x + b$.
Note: Instead of using the slope formula, you could reason that since the diagonal of a square forms two right triangles in which the vertical and horizontal sides always have the same length, their ratio is always 1. Since $\overline{AC}$ rises from left to right, its slope is positive. Therefore, the slope of $\overline{AC}$ is 1 (and the slope of diagonal $\overline{BD}$ is -1).
   - To find b, substitute the coordinates of $C(7,6)$ into $y = x + b$, which makes $6 = 7 + b$, so $b = -1$.
   - An equation of $\overline{AC}$ is $y = x - 1$.

24. (**B**) The general form of an equation of line p is $y = mx + b$. It is given that the slope of the line is $\frac{3}{2}$ and the line contains the point (6,2). Substitute $m = \frac{3}{2}$, $x = 6$, and $y = 2$ in the equation of the line and solve for b:

$$\begin{aligned} y &= mx + b \\ 2 &= \frac{3}{2}(6) + b \\ 2 &= 9 + b \\ -7 &= b \end{aligned}$$

GRID-IN

1. (**3/4**) Since the line that passes through points $(7,3k)$ and $(0,k)$ has a slope of $\frac{3}{14}$,

$$\begin{aligned} \frac{3k-k}{7-0} &= \frac{3}{14} \\ \frac{2k}{7} &= \frac{3}{14} \\ 28k &= 21 \\ k &= \frac{21}{28} = \frac{3}{4} \end{aligned}$$

Grid in as 3/4. .

2. (**3/2**) Since the slope of line ℓ_1 is $\frac{5}{6}$, then

$$\frac{y_1 - 0}{3-0} = \frac{5}{6} \quad \text{or} \quad y_1 = \frac{15}{6} = \frac{5}{2}$$

The slope of line ℓ_2 is $\frac{1}{3}$, so

$$\frac{y_2 - 0}{3-0} = \frac{1}{3} \quad \text{or} \quad y_2 = \frac{3}{3} = 1$$

Since points A and B have the same x-coordinates, they lie on the same vertical line, so

$$\begin{aligned} \text{Distance from } A \text{ to } B &= y_1 - y_2 \\ &= \frac{5}{2} - 1 \text{ or } \frac{3}{2} \end{aligned}$$

Grid in as 3/2.

3. (**13.5**) The area of the shaded region is equal to the difference in the areas of the two overlapping right triangles formed by the coordinate axes and the slanted lines:

$$\begin{aligned} \text{Area of shaded region} &= \frac{1}{2}(6)(5) - \frac{1}{2}(1)(3) \\ &= 15 - 1.5 \\ &= 13.5 \end{aligned}$$

4. (**5**) Find the value of the y-intercept "b" in the equation of the line that is perpendicular to $\overline{AB}$ at point M.
   - The slope of $\overline{AB}$ =

$$\frac{\Delta y}{\Delta x} = \frac{12-(-4)}{6-(-2)} = \frac{12+4}{6+2} = \frac{16}{8} = 2$$