YII 数学　SAT 演習

●6-7 Solid Figures

1. What is the volume of a cube whose surface area is 96?

(A) $16\sqrt{2}$
(B) 32
(C) 64
(D) 125
(E) 216

2. The length, width, and height of a rectangular solid are in the ratio of 3 : 2 : 1. If the volume of the box is 48, what is the total surface area of the box?

(A) 27
(B) 32
(C) 44
(D) 64
(E) 88

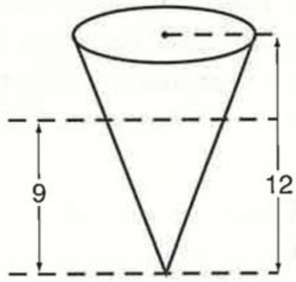
3. If X is the center point of a face of a cube with a volume of 8, and Y is the center point of the opposite face of this cube, what is the distance from X to Y?

(A) $\sqrt{2}$
(B) 2
(C) $2\sqrt{2}$
(D) 4
(E) 6

4. A cube whose volume is $\frac{1}{8}$ cubic foot is placed on top of a cube whose volume is 1 cubic foot. The two cubes are then placed on top of a third cube, whose volume is 8 cubic feet. What is the height, in *inches*, of the stacked cubes?

(A) 30
(B) 40
(C) 42
(D) 44
(E) 64

5.



Note: Figure is not drawn to scale.

The height of the solid cone above is 12 inches, and the area of the circular base is 64π square inches. What is the area, in *square inches,* of the base of the cone formed when a plane parallel to the base cuts through the original cone 9 inches above the vertex of the cone?

(A) 9π
(B) 16π
(C) 25π
(D) 36π
(E) 49π

6. If the height of a cylinder is doubled, by what number must the radius of the base be multiplied so that the resulting cylinder has the same volume as the original cylinder?

(A) 4
(B) 2
(C) $\frac{1}{\sqrt{2}}$
(D) $\frac{1}{2}$
(E) $\frac{1}{4}$

7. A rectangular fish tank has a base 2 feet wide and 3 feet long. When the tank is partially filled with water, a solid cube with an edge length of 1 foot is placed in the tank. If no overflow of water from the tank is assumed, by how many *inches* will the level of the water in the tank rise when the cube becomes completely submerged?

(A) $\frac{1}{6}$
(B) $\frac{1}{2}$
(C) 2
(D) 3
(E) 4

8. The volume of a cylinder of radius r is $\frac{1}{4}$ of the volume of a rectangular box with a square base of side length x. If the cylinder and the box have equal heights, what is r in terms of x?

(A) $\frac{x^2}{2\pi}$

(B) $\frac{x}{2\sqrt{\pi}}$

(C) $\frac{\sqrt{2x}}{\pi}$

(D) $\frac{\pi}{2\sqrt{x}}$

(E) $\sqrt{2\pi x}$

9. The height of sand in a cylinder-shaped can drops 3 inches when 1 cubic foot of sand is poured out. What is the diameter, in *inches*, of the cylinder?

(A) $\frac{2}{\sqrt{\pi}}$

(B) $\frac{4}{\sqrt{\pi}}$

(C) $\frac{16}{\pi}$

(D) $\frac{32}{\sqrt{\pi}}$

(E) $\frac{48}{\sqrt{\pi}}$

10. The height h of a cylinder equals the circumference of the cylinder. In terms of h, what is the volume of the cylinder?

(A) $\frac{h^3}{4\pi}$

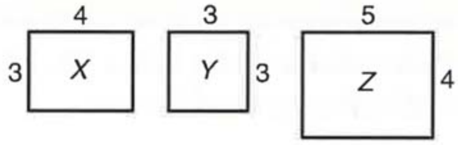
(B) $\frac{h^2}{2\pi}$

(C) $\frac{h^3}{2}$

(D) $h^2 + 4\pi$

(E) πh^3

11.



For which of the following combinations of rectangular faces X, Y, and Z in the figures above can a rectangular solid be formed?

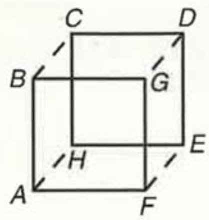
I. Two of face X, two of face Y, and two of face Z
II. Four of face X and two of face Y
III. Two of face Y and four of face Z

(A) I only
(B) II only
(C) I and III only
(D) II and III only
(E) None

12. A cylinder with radius r and height h is closed on the top and bottom. Which of the following expressions represents the total surface area of this cylinder?

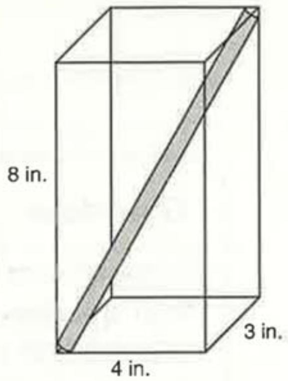
(A) $2\pi r(r + h)$
(B) $\pi r(r + 2h)$
(C) $\pi r(2r + h)$
(D) $\pi r^2 + 2h$
(E) $2\pi r^2 + h$

13.



In the figure above, if the edge length of the cube is 4, what is the shortest distance from A to D?

(A) $4\sqrt{2}$
(B) $4\sqrt{3}$
(C) 8
(D) $4\sqrt{2} + 4$
(E) $8\sqrt{3}$

14. 

A cylindrical tube with negligible thickness is placed into a rectangular box that is 3 inches by 4 inches by 8 inches, as shown in the accompanying diagram. If the tube fits exactly into the box diagonally from the bottom left corner to the top right back corner, what is the best approximation of the number of inches in the length of the tube?

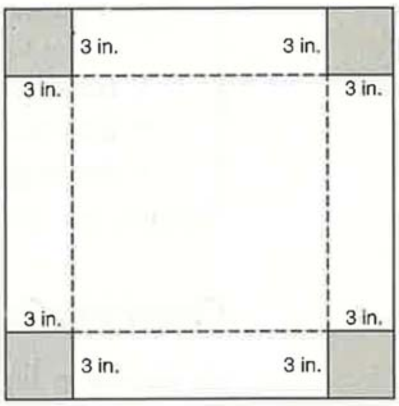
(A) 3.9
(B) 5.5
(C) 7.8
(D) 9.4
(E) 15.0

Grid-In

1. The dimensions of a rectangular box are integers greater than 1. If the area of one side of this box is 12 and the area of another side is 15, what is the volume of the box?

2. What is the number of inches in the minimum length of $\frac{1}{4}$-inch-wide tape needed to cover completely a cube whose volume is 8 cubic inches?

3. By what percent does the volume of a cube increase when the length of each of its sides is doubled?

4. 

Note: Figure is not drawn to scale.

A box is constructed by cutting 3-inch squares from the corners of a square sheet of cardboard, as shown in the accompanying diagram, and then folding the sides up. If the volume of the box is 75 cubic inches, find the number of square inches in the area of the *original* sheet of cardboard.

●6-7 Solid Figures 解答・解説

1. **(C)** A cube has six surfaces. If the surface area of the cube is 96, then the area of each square surface is $\frac{96}{6}$ or 16, so the edge length is $\sqrt{16}$ or 4. Hence, the volume of the cube is $4 \times 4 \times 4$ or 64.

2. **(E)** If the length, width, and height of a rectangular solid are in the ratio of 3 : 2 : 1, then let $3x$, $2x$, and x represent the dimensions of the solid. You are given that the volume of the box is 48. Hence:

$$(3x)(2x)(x) = 48$$
$$6x^3 = 48$$
$$x^3 = \frac{48}{6} = 8$$

The dimensions of the box are 2, 4, and 6 since $x = 2$, $2x = 2(2) = 4$, and $3x = 3(2) = 6$. The surface area of a rectangular box is 2 times the sum of the products of the three pairs of dimensions:

Surface area of box

$$\begin{aligned} &= 2\left[(\ell \times w) + (\ell \times h) + (h \times w)\right] \\ &= 2(2 \times 4 + 2 \times 6 + 4 \times 6) \\ &= 2(8 + 12 + 24) \\ &= 2(44) \\ &= 88 \end{aligned}$$

3. **(B)** If the volume of a cube is 8, then the edge length of the cube is 2 since $2 \times 2 \times 2 = 8$. The distance from point X at the center of a face of a cube to point Y at the center of the opposite face of the cube equals the edge length, which is 2.

4. **(C)** The height of the stacked cubes will be the sum of the edge lengths of the three cubes.
   - If the volume of the first cube is $\frac{1}{8}$ cubic feet, its edge length is $\frac{1}{2}$ of a foot or 6 inches.
   - If the volume of the second cube is 1 cubic foot, its edge length is 1 foot or 12 inches.
   - If the volume of the third cube is 8 cubic feet, its edge length is 2 feet or 2×12 or 24 inches.

   The height of the stacked cubes, in *inches*, is $6 + 12 + 24$ or 42.

5. **(D)** First find the radius of the circular base of the smaller cone formed when a plane parallel to the base cuts through the original cone.
   - Since the height of the original cone is 12 inches and the height of the smaller cone is 9 inches, the radius of the circular base of the smaller cone is $\frac{9}{12}$ or $\frac{3}{4}$ of the radius of the original cone.
   - You are given that the area of the original cone is 64π, so the radius of its base is $\sqrt{64}$ or 8. Hence, the radius of the circular base of the smaller cone is $\frac{3}{4} \times 8$ or 6.

   Since the radius of the base of the smaller cone is 6 inches, the area, in *square inches*, of the base is $\pi 6^2$ or 36π.

6. **(C)** The volume V of a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height. If h is replaced by $2h$ and the radius is multiplied by a constant k, so that the volume V does not change, then

$$V = \pi r^2 h = \pi (kr)^2 (2h)$$

so

$$\pi r^2 h = \pi (k^2 r^2)(2h)$$
$$1 = 2k^2$$
$$\frac{1}{2} = k^2$$
$$\frac{1}{\sqrt{2}} = k$$

7. **(C)** When a solid cube with an edge length of 1 foot is placed in the fish tank, it displaces a volume of water that is equal to the volume of the cube, which is $1 \times 1 \times 1$ or 1 cubic foot. If h represents the change in the number of feet in the height of the water in the fish tank, then

$$\text{Volume of displaced water} = 2 \times 3 \times h = 1$$
$$h = \frac{1}{6} \text{ foot}$$

Hence, the number of *inches* the level of the water in the tank will rise is $\frac{1}{6} \times 12$ or 2.

8. **(B)** Let h represent the equal heights of the cylinder and the rectangular box. Since you are given that the volume of the cylinder of radius r is $\frac{1}{4}$ of the volume of the rectangular box with a square base of side length x,

$$\overbrace{\pi r^2 h}^{\text{Volume of cylinder}} = \frac{1}{4}\overbrace{(x \cdot x \cdot h)}^{\text{Volume of box}}$$
$$\pi r^2 h = \frac{x^2}{4} h$$

$$r^2 = \frac{x^2}{4\pi}$$

$$r = \frac{\sqrt{x^2}}{\sqrt{4\pi}} = \frac{x}{2\sqrt{\pi}}$$

9. (**E**) You are given that the height of the sand in the cylinder-shaped can drops 3 inches or $\frac{1}{4}$ foot when 1 cubic foot of sand is poured out. If r represents the length in feet of the radius of the circular base, then the volume of the sand poured out must be equal to 1. Hence:

$$\text{Volume of sand} = \pi r^2\left(\frac{1}{4}\right) = 1$$

$$\pi r^2 = 1 \times 4$$

$$r^2 = \frac{4}{\pi}$$

$$r = \frac{\sqrt{4}}{\sqrt{\pi}} = \frac{2}{\sqrt{\pi}}\text{ feet}$$

$$= \frac{2}{\sqrt{\pi}}\text{ feet} \times 12\text{ inches/feet}$$

$$= \frac{24}{\sqrt{\pi}}\text{ inches}$$

Hence the radius of the cylinder is $\frac{2}{\sqrt{\pi}}$ feet $\times$ 12 or $\frac{24}{\sqrt{\pi}}$ inches, and the diameter in *inches* is $2 \times \frac{24}{\sqrt{\pi}}$ or $\frac{48}{\sqrt{\pi}}$.

10. (**A**) If the height h of a cylinder equals the circumference of the cylinder with radius r, then $h = 2\pi r$, so $r = \frac{h}{2\pi}$. Hence:

$$\text{Volume of cylinder} = \pi r^2 h$$

$$= \pi\left(\frac{h}{2\pi}\right)^2 h$$

$$= \pi\left(\frac{h^2}{4\pi^2}\right) h$$

$$= \frac{\pi h^3}{4\pi^2}$$

$$= \frac{h^3}{4\pi}$$

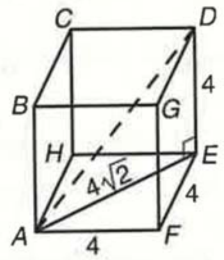
11. (**B**) For each Roman numeral combination determine whether a rectangular solid can be formed.

- I. If the set of six faces of a rectangular solid includes two of face X and two of face Y, then the rectangular solid must be a 3 by 3 by 4 solid. Since face Z is 4 by 5, it cannot be used to complete the rectangular solid. Hence, the six faces in combination I cannot be used to form a rectangular solid.
- II. If the set of six faces of a rectangular solid include four of face X, then these four faces can be arranged so that they form an open box with square 3 by 3 bases at opposite ends of the box. Since face Y is a 3 by 3 square, it can be used to complete the rectangular solid. Hence, the six faces in combination II can be used to form a rectangular solid.
- III. If the set of six faces of a rectangular solid includes four of face Z, then these four faces can be arranged so that they form an open box with either 4 by 4 squares or 5 by 5 squares at opposite ends of the box. Since face Y is a 3 by 3 square, it cannot be used to complete the rectangular solid. Hence, the six faces in combination III cannot be used to form a rectangular solid.

Only Roman numeral combination II gives faces from which a rectangular solid can be formed.

12. (**A**) If a cylinder with radius r and height h is closed on the top and bottom, then the sum of the areas of the circular top and bottom is $\pi r^2 + \pi r^2$ or $2\pi r^2$. The area of the curved surface is the height h times the distance around the curved surface, $2\pi r$. Hence, the total surface area is $2\pi r^2 + 2\pi rh$ or $2\pi r(r + h)$.

13. (**B**) *Solution 1.* The shortest distance from A to D is the length of AD. Draw AD, which is the hypotenuse of a right triangle whose legs are AE and ED. Since the edge length of the cube is given as 4, $ED = AF = 4$. Segment AE is the diagonal of square $AFEH$, so $AE = 4\sqrt{2}$.



In right triangle AED,

$$(AD)^2 = (AE)^2 + (ED)^2$$

$$= \left(4\sqrt{2}\right)^2 + (4)^2$$

$$= (16 \cdot 2) + 16$$

$$= 32 + 16$$

$$AD = \sqrt{48} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

Solution 2.

$$\begin{aligned} \text{Diagonal } d &= \sqrt{\ell^2 + w^2 + h^2} \\ \text{Let } \ell = w = h = 4: &= \sqrt{4^2 + 4^2 + 4^2} \\ &= \sqrt{16 + 16 + 16} \\ &= \sqrt{48} \\ &= \sqrt{16} \cdot \sqrt{3} \\ &= 4\sqrt{3} \end{aligned}$$

14. **(D)** Let $\ell = 4$, $w = 3$, $h = 8$:

$$\begin{aligned} \text{Diagonal } d &= \sqrt{\ell^2 + w^2 + h^2} \\ &= \sqrt{4^2 + 3^2 + 8^2} \\ &= \sqrt{16 + 9 + 64} \\ &= \sqrt{89} \\ &\approx 9.4 \end{aligned}$$

GRID-IN

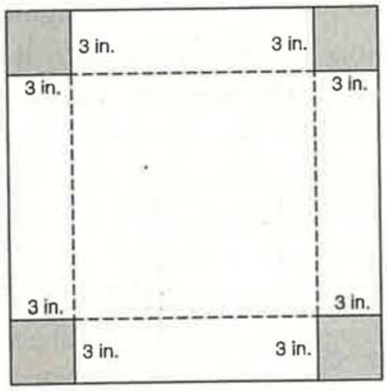
1. **(60)** You are given that all the dimensions of a rectangular box are integers greater than 1.

   Since the area of one side of this box is 12, the dimensions of this side must be either 2 by 6 or 3 by 4. The area of another side of the box is given as 15, so the dimensions of this side must be 3 by 5. Since the two sides must have at least one dimension in common, the dimensions of the box are 3 by 4 by 5, so its volume is $3 \times 4 \times 5$ or 60.
2. **(96)** A cube whose volume is 8 cubic inches has an edge length of 2 inches since $2 \times 2 \times 2 = 8$. Since a cube has six square faces of equal area, the surface area of this cube is 6×2^2 or 6×4 or 24. The minimum length L of $\frac{1}{4}$-inch-wide tape needed to completely cover the cube must have the same surface area as the cube. Hence, $L \times \frac{1}{4} = 24$ and $L = 24 \times 4 = 96$ inches.
3. **(700)** Pick an easy number for the edge length of the cube. If the edge length is 1, the volume of the cube is $1 \times 1 \times 1$ or 1. If the length of each side of this cube is doubled, a cube with an edge length of 2 results. The volume of the new cube is $2 \times 2 \times 2$ or 8. Hence:

$$\begin{aligned} \%\text{ increase in volume} &= \frac{\text{Increase in volume}}{\text{Original volume}} \times 100\% \\ &= \frac{8 - 1}{1} \times 100\% \\ &= 700\% \end{aligned}$$

Grid in as 700.

4. (121)



- Since 3-inch squares from the corners of the square sheet of cardboard are cut and folded up to form a box, the height of the box thus formed is 3 inches.
- If x represents the length of a side of the square sheet of cardboard, then the length and width of the box is $x - 6$.
- Since the volume of the box is 75 cubic inches:

$$\begin{aligned} \text{length} \times \text{width} \times \text{height} &= \text{Volume of box} \\ (x - 6)(x - 6)3 &= 75 \\ (x - 6)^2 &= \frac{75}{3} = 25 \\ x - 6 &= \sqrt{25} = 5 \\ x &= 5 + 6 = 11 \end{aligned}$$

Because the length of each side of the original square sheet of cardboard is 11 inches, the area of the square sheet of cardboard is $11 \times 11 = 121$ square inches.